A practical primal-dual interior-point algorithm for nonsymmetric conic optimization

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- The two nonsymmetric cones

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Summary

Section 1

Conic optimization

Generic conic optimization problem

Primal form

$$
\begin{array}{lrcl}
\text{minimize} & \sum_k (c^k)^T x^k & & \\
\text{st} & \sum_k A^k x^k & = & b, \\
 & x^k \in \mathcal{K}^k, & & \forall k,
\end{array}
$$

where

- $c^k \in \mathcal{R}^{n^k}$,
- $A^k \in \mathcal{R}^{m \times n^k}$,
- $b \in \mathcal{R}^m$,
- $\mathcal{K}^k$ are convex cones.

The 5 cones

3 symmatric cones:

- Linear.
- Quadratic.
- Semidefinite.

2 nonsymmetric cones:

- **Exponential**.
- **Power**.

Observation:

- **Almost** all convex optimization problems appearing in practice can be formulated using those 5 cones.

The power cone

The power cone:

$$\mathcal{K}_{pow}(\alpha) := \left\{ (x, z) : \prod_{j=1}^{n} x_j^{|\alpha_j|} \geq \|z\|^{\sum_{j=1}^{n} |\alpha_j|},\ x \geq 0 \right\}.$$

Examples ($\alpha \in (0,1)$):

$$\begin{aligned}
(t, 1, x) \in \mathcal{K}_{pow}(\alpha, 1-\alpha) \quad &\Leftrightarrow \quad t \geq |x|^{1/\alpha},\ t \geq 0, \\
(x, 1, t) \in \mathcal{K}_{pow}(\alpha, 1-\alpha) \quad &\Leftrightarrow \quad x^{\alpha} \geq |t|, x \geq 0, \\
(x, t) \in \mathcal{K}_{pow}(e) \quad &\Leftrightarrow \quad \left(\prod_{j=1}^{n} x_j \right)^{1/n} \geq |t|,\ x \geq 0.
\end{aligned}$$

See also Chares [2] and the Mosek modelling cookbook [5].

The exponential cone

The exponential cone

$$\mathcal{K}_{\exp} := \{(x_1, x_2, x_3) : \ x_1 \geq x_2 e^{\frac{x_3}{x_2}}, \ x_2 \geq 0\} \\ \cup \{(x_1, x_2, x_3) : \ x_1 \geq 0, x_2 = 0, \ x_3 \leq 0\}$$

Applications:

$$\begin{aligned} (t, 1, x) \in \mathcal{K}_{\exp} &\Leftrightarrow t \geq e^x, \\ (t, 1, \ln(a)x) \in \mathcal{K}_{\exp} &\Leftrightarrow t \geq a^x, \\ (x, 1, t) \in \mathcal{K}_{\exp} &\Leftrightarrow t \leq \ln(x), \\ (1, x, t) \in \mathcal{K}_{\exp} &\Leftrightarrow t \leq -x \ln(x), \\ (y, x, -t) \in \mathcal{K}_{\exp} &\Leftrightarrow t \geq x \ln(x/y), \text{(relative entropy)}. \end{aligned}$$

Geometric programming + many more [2, 5].

Section 2

A primal-dual interior-point algorithm

Survey

- Lesson learned from the linear case: Solve the primal and dual problem simultaneously.
- Symmetric cones: Employ the Nesterov-Todd (NT) algorithm [10, 12].
- Nonsymmetric cones: How to generalize the NT algorithm?
    - Nesterov [8, 9], Skajaa and Ye [15], Serrano [14]: Computational results available
    - Tuncel [16], Myklebust [6], Tuncel and Myklebust [7]: No computational results.

Present work:

- Follows Myklebust and Tuncel.

Primal and dual problem

Primal problem

$$\begin{array}{llll} \text{minimize} & c^T x & & \\ \text{st} & Ax & = & b, \\ & x \in \mathcal{K} & & \end{array}$$

and the dual

$$\begin{array}{llll} \text{maximize} & b^T y & & \\ \text{st} & A^T y + s & = & c, \\ & s \in (\mathcal{K})^* & & \end{array}$$

where

$$\mathcal{K} = \mathcal{K}^1 \times \mathcal{K}^2 \times \cdots \mathcal{K}^k$$

and

$$\mathcal{K}^*$$

is the corresponding dual cone. Known for the 5 cone types.

Barrier functions

Define a 3 times differentiable function F such that

$$F : \text{int}(K) \mapsto \mathcal{R}$$

then it is a ν-logarithmically homogeneouos self-concordant barrier (ν-LHSCB) for int(K) if

$$|F'''(x)[u, u, u]| \leq 2(F''(x)[u, u])^{3/2}$$

and

$$F(\tau x) = F(x) - \nu \log \tau.$$

See [10, 12].

The dual barrier

If F is a ν-self-concordant barrier for K, then the Fenchel conjugate

$$F_*(s) = \sup_{x \in \mathrm{int}(K)} \{-\langle s, x\rangle - F(x)\}. \tag{1}$$

is a ν-self-concordant barrier for K^*. Let

$$\tilde{x} := -F_*'(s), \quad \tilde{s} := -F'(x), \quad \mu := \frac{\langle x, s\rangle}{\nu}, \quad \tilde{\mu} := \frac{\langle \tilde{x}, \tilde{s}\rangle}{\nu}.$$

Then $\tilde{x} \in \mathrm{int}(K)$, $\tilde{s} \in \mathrm{int}(K^*)$ and

$$\mu\tilde{\mu} \geq 1 \tag{2}$$

with equality iff $x = -\mu\tilde{x}$ (and $s = \mu\tilde{s}$).

The homogeneous model

Generalized Goldman-Tucker homogeneous model:

$$
(H) \qquad \begin{array}{rcl}
Ax - b\tau & = & 0, \\
A^T y + s - c\tau & = & 0, \\
-c^T x + b^T y - \kappa & = & 0, \\
(x;\tau) \in \bar{\mathcal{K}}, (s;\kappa) \in \bar{\mathcal{K}}^* & &
\end{array}
$$

where

$$
\bar{\mathcal{K}} := \mathcal{K} \times \mathcal{R}_+ \quad \text{and} \quad \bar{\mathcal{K}}^* := \mathcal{K}^* \times \mathcal{R}_+.
$$

- $\mathcal{K}$ is Cartesian product of $k+1$ convex cones.
- The homogeneous model always has a solution.
- Partial list of references:
  - Linear case: [4], [3], [17].
  - Nonlinear case: [11].

Investigating the homogeneous model

Lemma

Let $(x^*, \tau^*, y^*, s^*, \kappa^*)$ *be any feasible solution to (H), then*

i)
$$(x^*)^T s^* + \tau^* \kappa^* = 0.$$

ii) If $\tau^* > 0$*, then*
$$(x^*, y^*, s^*)/\tau^*$$
is an optimal solution.

iii) If $\kappa^* > 0$*, then at least one of the strict inequalities*
$$b^T y^* > 0 \tag{3}$$
and
$$c^T x^* < 0 \tag{4}$$
holds. If the first inequality holds, then (P) *is infeasible. If the second inequality holds, then* (D) *is infeasible.*

The central path

The central path:

$$\begin{aligned} Ax - b\tau &= \gamma(A\hat{x} - b\hat{\tau}), \\ A^T y + s - c\tau &= \gamma(A^T\hat{y} + \hat{s} - c\hat{\tau}), \\ -c^T x + b^T y - \kappa &= \gamma(-c^T\hat{x} + b^T\hat{y} - \hat{\kappa}), \\ s + \gamma\hat{\mu}F'(x) &= 0, \\ \tau\kappa - \gamma\hat{\mu} &= 0, \end{aligned}$$

where

$$\hat{\mu} := \frac{(\hat{x})^T\hat{s} + \hat{\tau}\hat{\kappa}}{\nu + 1}$$

and $(\hat{x}, \hat{\tau}, \hat{y}, \hat{s}, \hat{\kappa})$ is an "interior" solution for $\gamma = 1$. The central path is the solutions parameterised by $\gamma \in [0, 1]$.

Tracing the central path

- Idea: Trace the central path using Newton's method.
- Question: Should we use the primal or dual barrier i.e.

$$s + \gamma\hat{\mu}F'(x) = s + \gamma\hat{\mu}\tilde{s} = 0$$

or

$$x + \gamma\hat{\mu}F'_*(s) = x + \gamma\hat{\mu}\tilde{x} = 0$$

where

$$\tilde{x} := -F'_*(s) \text{ and } \tilde{s} := -F'(x).$$

Primal-dual scaling

A nonsingular matrix W is called a primal-dual scaling if it satisfies

$$\begin{array}{rcccl} v & := & Wx & = & W^{-T}s, \\ \tilde{v} & := & W\tilde{x} & = & W^{-T}\tilde{s}. \end{array}$$

The primal or dual centrality conditions are equivalent to

$$v = \gamma\hat{\mu}\tilde{v}.$$

- Result: The centrality conditions have become symmetric!

The search direction

Affine direction:

$$
\begin{aligned}
Ad_x^a - bd_\tau^a &= -(Ax^0 - b\tau^0), \\
A^T d_y^a + d_s^a - cd_\tau^a &= -(A^T y^0 + s^0 - c\tau^0), \\
-c^T d_x^a + b^T d_y^a - d_\kappa^a &= -(-c^T x^0 + b^T y^0 - \kappa^0), \\
W d_x^a + W^{-T} d_s^a &= -v^0, \\
\tau^0 d_\tau^a + \kappa^0 d_\tau^a &= -\tau^0 \kappa^0.
\end{aligned}
$$

Centering direction:

$$
\begin{aligned}
Ad_x^c - bd_\tau^c &= (Ax^0 - b\tau^0), \\
A^T d_y^c + d_s^a - cd_\tau^c &= (A^T y^0 + s^0 - c\tau^0), \\
-c^T d_x^c + b^T d_y^c - d_\kappa^c &= (-c^T x^0 + b^T y^0 - \kappa^0), \\
W d_x^c + W^{-T} d_s^c &= \mu^0 \tilde{v}^0, \\
\tau^0 d_\tau^c + \kappa^0 d_\tau^c &= \mu^0.
\end{aligned}
$$

Updating the solution

For a given $\gamma \in [0,1]$ then define

$$\begin{aligned} d_x &:= d_x^a + \gamma d_x^c, \\ d_\tau &:= d_\tau^a + \gamma d_\tau^c, \\ d_y &:= d_y^a + \gamma d_y^c, \\ d_s &:= d_s^a + \gamma d_s^c, \\ d_\kappa &:= d_\kappa^a + \gamma d_\kappa^c, \end{aligned}$$

and hence for a step size $\alpha \in [0,1]$ we have

$$\begin{aligned} x^+ &:= x^0 + \alpha d_x, \\ \tau^+ &:= \tau^0 + \alpha d_\tau, \\ y^+ &:= y^0 + \alpha d_y, \\ s^+ &:= s^0 + \alpha d_s, \\ \kappa^+ &:= \kappa^0 + \alpha d_\kappa. \end{aligned}$$

Basic but important properties

$$\begin{array}{rcl} Ax^+ - b\tau^+ & = & (1-\alpha(1-\gamma))(Ax^0 - b\tau^0), \\ A^T y^+ + s^+ - c\tau^+ & = & (1-\alpha(1-\gamma))(A^T y^0 + s^0 - c\tau^0), \\ -c^T x^+ + b^T y^+ - \kappa^+ & = & (1-\alpha(1-\gamma))(-c^T x^0 + b^T y^0 - \kappa^0), \\ (x^+)^T (s^+) + \tau^+ \kappa^+ & = & (1-\alpha(1-\gamma))((x^0)^T s^0 + \tau^0 \kappa^0). \end{array}$$

- **Equal** decrease in infeasibility and complementarity for $\gamma \in [0, 1)$.
- If $\alpha \in]0, 1]$, then "convergence".
- No merit function is needed. Yahooooo!

Choice of the primal-dual scaling

Our method inspired by (Tuncel, Tuncel and Myklebust):

$$\begin{aligned} W^T W &\approx \mu^0 F''(x^0), \\ Wx &= W^{-T} s, \\ W\tilde{x} &= W^{-T}\tilde{s}. \end{aligned}$$

Employ the quasi Newton idea to compute W.

Computing the scaling matrix

Theorem (Schnabel [13])

Let $\bar{X}, \bar{S} \in \mathcal{R}^{n \times p}$ have full rank p. Then there exists $H \succ 0$ such that $H\bar{X} = \bar{S}$ if and only if $\bar{S}^T \bar{X} \succ 0$.

As a consequence

$$H = \bar{S}(\bar{S}^T \bar{X})^{-1} \bar{S}^T + Z Z^T$$

where $\bar{X}^T Z = 0$, rank$(Z) = n - p$. We have $n = 3$, $p = 2$ and

$$\bar{X} := \begin{pmatrix} x & \tilde{x} \end{pmatrix}, \quad \bar{S} := \begin{pmatrix} s & \tilde{s} \end{pmatrix},$$

with

$$\det(\bar{S}^T \bar{X}) = \nu^2(\mu\tilde{\mu} - 1) \geq 0$$

vanishing only on the central path.

Computing the scaling matrix

Any scaling with $n = 3$ satisfies

$$W^T W = \bar{S}(\bar{S}^T \bar{X})^{-1} \bar{S}^T + z z^T$$

where $\begin{pmatrix} x & \tilde{x} \end{pmatrix}^T z = 0$, $z \neq 0$. Expanding the BFGS update [13]

$$H^+ = H + \bar{S}(\bar{S}^T \bar{X})^{-1} \bar{S}^T - H\bar{X}(\bar{X}^T H \bar{X})^{-1} \bar{X}^T H,$$

for $H \succ 0$ gives the scaling by Tunçel [16] and Myklebust [7], i.e.,

$$z z^T = H - H\bar{X}(\bar{X}^T H \bar{X})^{-1} \bar{X}^T H,$$

with $H = \mu F''(x)$.

A high-order corrector term

A high-order correction:

$$\begin{aligned}
Ad_x^{co} - bd_\tau^{co} &= 0,\\
A^T d_y^{co} + d_s^{co} - cd_\tau^{co} &= 0,\\
-c^T d_x^{co} + b^T d_y^{co} - d_\kappa^{co} &= 0,\\
W d_x^{co} + W^{-T} d_s^{co} &= -\frac{1}{2} W^{-T} F'''(x)[d_x^a, F''(x)^{-1} d_s^a],\\
\tau^0 d_\kappa^{co} + \kappa^0 d_\tau^{co} &= -d_\tau^a d_\kappa^a.
\end{aligned}$$

For motivation see paper.
Finally

$$\begin{aligned}
d_x &:= d_x^a + \gamma d_x^c + d_x^{co},\\
d_\tau &:= d_\tau^a + \gamma d_\tau^c + d_\tau^{co},\\
d_y &:= d_y^a + \gamma d_y^c + d_y^{co},\\
d_s &:= d_s^a + \gamma d_s^c + d_s^{co},\\
d_\kappa &:= d_\kappa^a + \gamma d_\kappa^c + d_\kappa^{co}.
\end{aligned}$$

The power cone case

A 3-self-concordant barrier for the 3 dimensional primal power cone:

$$F(x) = -\log(x_1^{2\alpha} x_2^{2-2\alpha} - x_3^2) - (1-\alpha)\log x_1 - \alpha \log x_2. \quad (5)$$

suggest by Chares [2]. Generalized in [1].
Is self-dual using redefined inner product.
However,

- The conjugate barrier $F_*(x)$ or its derivatives cannot be evaluated on closed-form.
- Can be evaluated numerically to high accuracy based of an idea of Nesterov.

The exponential cone case

A 3-self-concordant barrier for the primal exponential cone:

$$F(x) = -\log(x_2 \log(x_1/x_2) - x_3) - \log x_1 - \log x_2. \quad (6)$$

The dual exponential cone is

$$\mathcal{K}_e^* = \mathrm{cl}\{z \in \mathcal{R}^3 \mid e \cdot z_1 \geq -z_3 \exp(z_2/z_3),\ z_1 > 0, z_3 < 0\}. \quad (7)$$

However,

- The conjugate barrier $F_*(x)$ or its derivatives cannot be evaluated on closed-form.
- Can be evaluated numerically to high accuracy based of an idea of Nesterov.

The starting point

Easy to compute $(\hat{x}, \hat{s})$ such that

$$\hat{s} + F'(\hat{x}) = 0.$$

Next compute

$$\rho = \sqrt{\min((1.0 + \|c\|)(1.0 + \|b\|), 1e6)}$$

then let

$$\begin{aligned} x^0 &= \rho\hat{x}, \\ \tau^0 &= \rho, \\ y^0 &= 0, \\ s^0 &= \rho\hat{s}, \\ \kappa^0 &= \rho. \end{aligned}$$

Computing the search direction

- First compute the affine direction and then compute γ.
- Second compute final direction with centering and corrector.
- Reuse matrix factorization of the Newton equations for the two solves.

Step size computations

Define the one sided neighborhood

$$\mathcal{N}(\beta) = \left\{ (x, s, \tau, \kappa) \mid \tau\kappa \geq \beta\mu,\ \nu_i \langle F'(x_i), F'_*(s_i) \rangle^{-1} \geq \beta\mu,\ \forall i \right\},$$

for some $\beta \in]0, 1]$. This bounds

$$\mu\tilde{\mu} \geq 1$$

from above.
We use bisection to compute a step size $\alpha \in]0, 1]$ so

- New point is feasible
- and in the neighborhood.

The termination criteria

Let

$$(x^k, y^k, s^k, \tau^k, \kappa^k)$$

the k'th interior-point iterate to the homogeneous model. Since,

$$x^k \in K, s^k \in K^* \text{ and } \tau^k, \kappa^k > 0$$

then compute

$$\rho_p^k = \min\left\{\rho \mid \left\|A\frac{x^k}{\tau^k} - b\right\|_\infty \leq \rho\varepsilon_p(1 + \|b\|_\infty)\right\},$$

$$\rho_d^k = \min\left\{\rho \mid \left\|A^T\frac{y^k}{\tau^k} + \frac{s^k}{\tau^k} - c\right\|_\infty \leq \rho\varepsilon_d(1 + \|c\|_\infty)\right\},$$

$$\rho_g^k = \min\{ \quad \rho \mid \min\left(\frac{(x^k)^T s^k}{(\tau^k)^2}, \left|\frac{c^T x^k}{\tau^k} - \frac{b^T y^k}{\tau^k}\right|\right) \leq \rho\varepsilon_g \max\left(1, \frac{\min\left(\left|c^T x^k\right|, \left|b^T y^k\right|\right)}{\tau^k}\right)\},$$

$$\rho_{pi}^k = \min\left\{\rho \mid \left\|A^T y^k + s^k\right\|_\infty \leq \rho\varepsilon_i b^T y^k,\ b^T y^k > 0\right\},$$

$$\rho_{di}^k = \min\left\{\rho \mid \left\|A x^k\right\|_\infty \leq -\rho\varepsilon_i c^T x^k,\ c^T x^k < 0\right\},$$ and

$$\rho_{ip}^k = \min\left\{\rho \mid \left\| \begin{matrix} A^T y^k + s^k \\ A x^k \end{matrix} \right\|_\infty \leq \rho\varepsilon_i \left\| \begin{matrix} y^k \\ s^k \\ x^k \end{matrix} \right\|_\infty, \left\| \begin{matrix} y^k \\ s^k \\ x^k \end{matrix} \right\|_\infty > 0\right\}.$$

Optimality certificate

Note

$$\varepsilon_p, \varepsilon_d, \varepsilon_g, \varepsilon_i$$

are nonnegative user specified tolerances.
Observe if for instance

$$\max(\rho_p^k, \rho_d^k, \rho_g^k) \leq 1$$

then

$$\begin{aligned}
\left\| A \frac{x^k}{\tau^k} - b \right\|_\infty &\leq \varepsilon_p (1 + \|b\|_\infty), \\
\left\| A^T \frac{y^k}{\tau^k} + \frac{s^k}{\tau^k} - c \right\|_\infty &\leq \varepsilon_d (1 + \|c\|_\infty), \\
\min\left(\frac{(x^k)^T s^k}{(\tau^k)^2}, |\frac{c^T x^k}{\tau^k} - \frac{b^T y^k}{\tau^k}| \right) &\leq \varepsilon_g \max\left(1, \frac{\min\left(\left|c^T x^k\right|, \left|b^T y^k\right| \right)}{\tau^k}\right)
\end{aligned}$$

and hence $(x^k, y^k, s^k))/\tau^k$ is an almost primal and dual feasible solution with small complementarity gap.

Primal infeasibility certificate

If

$$\rho_{pi} \leq 1$$

then

$$\left\| A^T y^k + s^k \right\|_\infty \leq \varepsilon_i b^T y^k,\ b^T y^k > 0$$

and define

$$\bar{y} := \frac{y^k}{b^T y^k} \text{ and } \bar{s} := \frac{s^k}{b^T y^k}$$

and hence

$$\begin{array}{rcl} b^T \bar{y} & \geq & 1, \\ \left\| A^T \bar{y} + \bar{s} \right\| & \leq & \varepsilon_i, \\ \bar{s} & \in & K^* \end{array}$$

i.e. an approximate certificate of primal infeasibility has been computed.

Dual infeasibility certificate

If

$$\rho_{di} \leq 1$$

then an approximate certificate of dual infeasibility has been computed.

Ill-posed certificate

Ifq

$$\rho_{ip} \leq 1$$

then

$$\left\| \begin{array}{c} A^T y^k + s^k \\ A x^k \end{array} \right\|_\infty \leq \varepsilon_i \left\| \begin{array}{c} y^k \\ s^k \\ x^k \end{array} \right\|_\infty, \left\| \begin{array}{c} y^k \\ s^k \\ x^k \end{array} \right\|_\infty > 0$$

i.e. an approximate certificate of ill-posedness.
Observe if for instance $\left\|y^k\right\|_\infty \gg 0$ then a tiny perturbation in b will make the problem infeasible. Try add $\varepsilon y^k / \left\|y^k\right\|_\infty$ to b.

Implemention

The algorithm has been integrated in Mosek v9.

- Problems is dualized if believed beneficial for lin. alg.
- A presolve is performed.
- A scaling is performed.
- Symmetric cones are handled with NT scaling.
- Simple bounds and fixed variables are handled efficiently in the linear algebra.
- Employs highly tuned linear algebra to solve the Newton equations.

Section 3

Computational results

Exponential/power cone optimization

- Hardware: AMD EPYC 7402P 2.80GHz, 24C/48T, 128M Cache (180W) (virum).
- MOSEK: Version 9.2.21.
- Threads: 8 threads is used in test to simulate a typical user environment.
- All timing results t are in wall clock seconds.
- Test problems: Public (e.g `cblib.zib.de`) and customer supplied.

Optimized problems

Name	# con.	# cone	# var.	# mat. var.
C_Table5_POW_T12	1086456	1328602	3991646	0
WassersteinReg_2img	972	1229312	3687936	0
dump_Prostate_VMAT_308	26872	14649	126320	0
logistic_mip-8-1000-bayes	326	237	1007	0
c-diaz_test_c47	164404	160000	519810	0
x20565-3over2pow	7932	2482	20320	0
z17926	2	75000	225000	0
task_dopt16	1600	26	376	2
entolib_ento2	26	4695	14085	0
entolib_a_56	37	9702	29106	0
exp-ml-scaled-20000	19999	20000	79998	0
exp-ml-20000	19999	20000	79998	0
patil3_conv	418681	413547	1264340	0
c-diaz_test_c47	164404	160000	519810	0
utkarsh_robust_29012019	1228800	819201	2867301	0
varun_conv	333	328	1009	0
z19841	160767	160766	483856	0
z19502	2354679	524286	6546535	0
udomsak	97653	97653	294519	0
relentr25000	1	25000	75000	0
cbf_mra02	3739	3620	11105	0
log-utility-200-5000	10003	5001	25206	0
cbf_cx02-100	5247	5149	15645	0
elmore_delay_16_conv	830	762	2375	0
gp_dave_3_conv	568	373	2052	0
fsparc_6_075_10	942	432	1806	0
c-260209-1	2238	1424	10079	0

Exponential/power cone optimization
Result

Name	P. obj.	# sig. fig.	# iter	time(s)
C_Table5_POW_T12	5.3368368447e-02	7	29	172.0
WassersteinReg_2img	-1.9199622575e+01	6	76	255.7
dump_Prostate_VMAT_308	5.6828282708e+04	8	58	752.9
logistic_mip-8-1000-bayes	6.6082723985e+01	9	42	0.1
c-diaz_test_c47	1.8880303425e-02	9	69	78.2
x20565-3over2pow	2.3137374801e+02	8	34	1.7
z17926	1.7942114183e-01	9	45	7.7
task_dopt16	1.3214504598e+01	9	12	0.6
entolib_ento2	-1.1354764143e+01	9	31	0.3
entolib_a_56	-8.2834853287e+00	8	89	4.0
exp-ml-scaled-20000	-3.3125859649e+00	9	139	11.0
exp-ml-20000	-1.9795438202e+04	11	110	4.6
patil3_conv	-1.0539216061e+00	9	88	115.7
c-diaz_test_c47	1.8880303425e-02	9	69	78.2
utkarsh_robust_29012019	1.6172995191e+00	7	83	699.4
varun_conv	-2.3527295782e+01	9	50	0.2
z19841	-2.6100556412e+00	9	85	275.6
z19502	5.1527196039e+06	10	68	398.2
udomsak	7.6466584697e-02	10	206	616.5
relentr25000	6.3511190583e-02	9	22	1.3
cbf_mra02	4.3179836838e+00	9	170	3.4
log-utility-200-5000	-1.8520357724e+03	9	27	1.5
cbf_cx02-100	7.7292742788e+00	8	21	0.7
elmore_delay_16_conv	4.6571224035e+00	8	26	0.2
gp_dave_3_conv	6.1849199940e+00	9	27	0.1
fsparc_6_075_10	4.6989895076e+02	9	20	0.0
c-260209-1	-6.8419351593e-02	9	38	4.1

Section 4

Summary

Summary

- Presented a primal-dual algorithm for nonsymmetric conic optimization.
- Makes it easy to extend the Nesterov-Todd algorithm to nonsymmetric cones.
- No polynomial complexity proof is provided.
- Good computational results are presented(only 3 dimensional though!).
- Possible improvements:
    - Handle more cone types.
    - Better primal-dual scaling.
    - High accuracy computations in scaling matrix computations.
    - Achieve fast convergence.
    - A multiple corrector.

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