Introduction to Mosek

Modern Optimization in Energy, 28 June 2018

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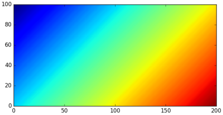
www.mosek.com

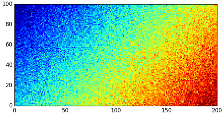
MOSEK package overview

- Started in 1999 by Erling Andersen
- Convex conic optimization package + MIP
- LP, QP, SOCP, SDP, other nonlinear cones
- Low-level optimization API
  - C, Python, Java, .NET, Matlab, R, Julia
- Object-oriented API *Fusion*
  - C++, Python, Java, .NET
- 3rd party
  - GAMS, AMPL, CVXOPT, CVXPY, YALMIP, PICOS, GPkit
- Conda package, .NET Core package
- Upcoming v9

Example: 2D Total Variation

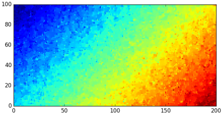
Someone sends you the left signal but you receive noisy f (right):

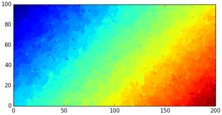


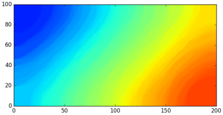


How to denoise/smoothen out/approximate u?

$$\begin{array}{ll} \text{minimize} & \sum_{ij}(u_{i,j} - u_{i+1,j})^2 + \sum_{ij}(u_{i,j} - u_{i,j+1})^2 \\ \text{subject to} & \qquad\qquad \sum_{ij}(u_{i,j} - f_{i,j})^2 \quad \leq \quad \sigma. \end{array}$$







Conic problems

A conic problem in canonical form:

$$\begin{array}{rc} \min & c^T x \\ \text{s.t.} & Ax + b \in \mathcal{K} \end{array}$$

where $\mathcal{K}$ is a product of cones:

- linear:

$$K = \mathbb{R}_{\geq 0}$$

- quadratic:

$$K = \{x \in \mathbb{R}^n \ : \ x_1 \geq \sqrt{x_2^2 + \cdots + x_n^2}\}$$

- semidefinite:

$$K = \{X \in \mathbb{R}^{n \times n} \ : \ X = FF^T\}$$

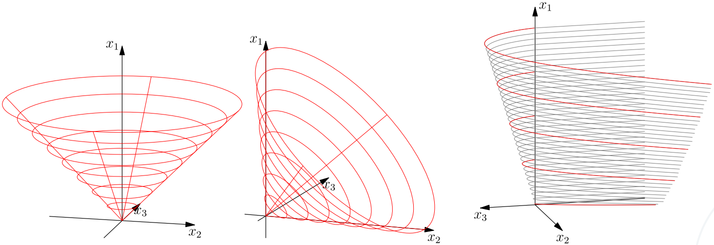
Conic problems, cont.

- exponential cone:

$$K = \{x \in \mathbb{R}^3 \;:\; x_1 \geq x_2 \exp(x_3/x_2),\; x_2 > 0\}$$

- power cone:

$$K = \{x \in \mathbb{R}^3 \;:\; x_1^{p-1} x_2 \geq |x_3|^p,\; x_1, x_2 \geq 0\},\; p > 1$$



$x_1 \geq \sqrt{x_2^2 + x_3^2}$, $2x_1x_2 \geq x_3^2$ $\qquad$ $x_1 \geq x_2 \exp(x_3/x_2)$

Conic representability

Lots of functions and constraints are representable using these cones.

$$|x|,\ \|x\|_1,\ \|x\|_2,\ \|x\|_\infty,\ \|Ax+b\|_2 \le c^T x + d$$

$$xy \ge z^2,\ x \ge \frac{1}{y},\ x \ge y^p,\ t \ge \left(\sum_i |x_i|^p\right)^{1/p} = \|x\|_p$$

$$t \le \sqrt{xy},\ t \le (x_1 \cdots x_n)^{1/n}, \text{geometric programming (GP)}$$

$$t \le \log x,\ t \ge e^x,\ t \le -x \log x,\ t \ge \log \sum_i e^{x_i},\ t \ge \log\left(1 + \frac{1}{x}\right)$$

$$\det(X)^{1/n},\ t \le \lambda_{min}(X),\ t \ge \lambda_{max}(X)$$

$$\text{convex } (1/2)x^T Q x + c^T x + q$$

Challenge

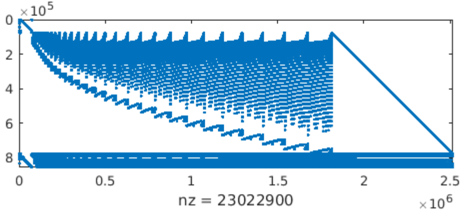
Find a

- natural,
- practical,
- important,
- convex

optimization problem, which cannot be expressed in conic form.

Optimizers in MOSEK

- primal simplex and dual simplex for linear problems
- primal-dual interior point method optimizer for all conic problems
- automatic dualization
- QPs transformed to conic quadratic form
- branch and bound and cut integer optimizer
- **exploits sparsity**



Example (DC-OPF)

$$
\begin{aligned}
\min \quad & \sum c_i P_{G_i} \\
\text{s.t.} \quad & P_{G_i}^{min} \leq P_{G_i} \leq P_{G_i}^{max} \\
& \mathbf{B} \cdot \theta = \mathbf{P_G} - \mathbf{P_D} \\
& |\theta_i - \theta_j| / x_{ij} \leq P_{ij,max}
\end{aligned}
$$

```python
def dcopf(ng, nb, PGmin, PGmax, Pmax, B, x, PD, c):
  M = Model()
  PG = M.variable(ng, Domain.inRange(PGmin, PGmax))
  theta = M.variable(nb)

  M.constraint(Expr.sub(Expr.mul(B, theta), Expr.sub(PG, PD)),
               Domain.equalsTo(0.0))
  M.constraint(Expr.sub(Var.hrepeat(theta,nb),
                        Var.vrepeat(theta.transpose(),nb)),
               Domain.lessThan(Pmax*x))

  M.objective(ObjectiveSense.Minimize, Expr.dot(c, PG))
```

Example (UCP)

A basic version of the Unit Commitment Problem with:

- quadratic generation costs,
- ramp constraints,
- minimal uptime and downtime,
- startup costs,

as an example of MISOCP. See:

→ `https://mosek.com/documentation`

  → The MOSEK notebook collection

    → Unit commitment

Contact us

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- Trial 30-day license
- Personal academic license
- Group academic license
- Commercial licenses

More:

- `www.mosek.com`
- `https://github.com/MOSEK/Tutorials`
- `https://themosekblog.blogspot.com/`
- `support@mosek.com`
- Modeling Cookbook `www.mosek.com/documentation`

Thank you!