Recent Advances In The New Mosek 8

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Improvements in version 8

An overview

- Presolve
  - Reduce memory requirements in the eliminator.
  - Conic presolve.
- Conic optimizer
  - Improve numerical stability particularly for SDOs.
  - Added a (auto) dualizer for conic quadratic problems.
- Convex quadratic optimizer
  - Use the conic optimizer internally.
- Mixed-integer optimizer.
  - Improved performance.
- Fusion.
  - Added a C++ version.
  - A lot of polishing and bug fixing.
- Optimization server.
- Conda availability.

Conic optimizer

- More aggressive problem scaling.
- Reworked formulas.
- Improved stopping criterion.
- Improved linear algebra.
- Automatic and transparent dualizer (not available for SDOs yet).
- Better ill posed problem handling e.g.

$$\begin{array}{ll} \text{minimize} & x_3 \\ \text{subject to} & x_1 = 0, \\ & 2x_1x_2 \geq x_3^2. \end{array}$$

Convex quadratic optimizer

Convex quadratic optimization model

$$\begin{aligned} \text{minimize} \quad & \frac{1}{2} x^T Q_0^T x + c^T x \\ \text{subject to} \quad & \frac{1}{2} x^T Q_i^T x + a_{i:} x \quad \leq \quad b_i, \quad i = 1 \ldots, m. \end{aligned} \tag{QO}$$

where:

- Symmetry: $Q_i = Q_i^T, \quad i =, \ldots, m.$
- Convexity: $Q_i \succeq 0.$

Hence, Q_i should be **positive semidefinite.**

Conic reformulation

MOSEK v8 convert (QO) to a (CQO) internally:

$$\begin{array}{lll} \text{minimize} & t_0 + c^T x & \\ \text{subject to} & t_i + a_{i:} x & = \quad b_i, \quad i =, 1 \ldots, m, \quad \text{(CQO)} \\ & L_i^T x - f_i & = \quad 0, \\ & z_i & = \quad 1, \\ & 2 z_i t_i \geq \|f_i\|^2 . & \end{array}$$

- Conversion to conic form is transparent.
- Leads to faster and more robust solution.
- Makes solution of convex quadratic problem run-to-run deterministic when multiple threads is employed.

The mixed integer optimizer

- Improved presolve.
- Presolve repeated after cutting plane generation if the optimizer deems it worthwhile.
- Improved heuristics.
- New cutting planes: Knapsack cover cuts.

Fusion: A tool for extreme disciplined modelling

Example:

$$
\begin{array}{lccl}
\text{maximize} & \mu^T x & & \\
\text{subject to} & e^T x & = & w + e^T x^0, \\
& [\gamma; G^T x] & \in & \mathcal{K}_q^{n+1}, \\
& x & \geq & 0.
\end{array}
$$

Python example

```
def BasicMarkowitz(n,mu,GT,x0,w,gamma):
    with  Model("Basic Markowitz") as M:

        # Redirect log output from the solver to stdout for debugging.
        # if uncommented.
        M.setLogHandler(sys.stdout)

        # Defines the variables (holdings). Shortselling is not allowed.
        x = M.variable("x", n, Domain.greaterThan(0.0))

        #  Maximize expected return
        M.objective('obj', ObjectiveSense.Maximize, Expr.dot(mu,x))

        # The amount invested  must be identical to initial wealth
        M.constraint('budget', Expr.sum(x), Domain.equalsTo(w+sum(x0)))

        # Imposes a bound on the risk
        M.constraint('risk', Expr.vstack( gamma,Expr.mul(GT,x)),
                     Domain.inQCone())

        M.solve()
```

Fusion improvements

General:

- Bug fixes.
- Polishing.
- Improved expression generators and performance.
- Support for C++ on Linux and MAC OS.
- Windows C++ support is waiting for a bug fix in MSC.

Python:

- Support for Python 3.4+ and Pypy/2.7.
- Use of Numpy arrays are default.

The optimization server

- Solve problems remotely with MOSEK using the optimization server.
- Submit problems, get solution through the web interface.

Python Anaconda availability

- Install the latest MOSEK using Anaconda
  - `conda install -c mosek mosek=8.0.43`
- Support for Anaconda Python 2.7, 3.4, and 3.5

Benchmark results

- Hardware: Intel based server.
- Software: Linux. MOSEK v7.1.0.58 and v8.0.0.42.
- Threads: 8 threads is used in test to simulate a typical user environment.
- All timing results t are in wall clock seconds.
- A small problem: Fastest optimizer has $t \leq 6$.
- A medium problem: Fastest optimizer has $t \leq 60$.
- Test problems: Public (e.g `cblib.zib.de`) and customer supplied.

For each problem we compute

$$r = \frac{t_8 + 0.01}{t_7 + 0.01}$$

where t_8 and t_7 is taken to solve a problem for version 8 and 7 respectively. The geometric average of the rs is shown.

Conic quadratic problems

	small		medium		large		fails'	
	7	8	7	8	7	8	7	8
Num.	887	887	127	127	30	30	37	18
Firsts	579	535	20	107	2	28		
Total time	1985.914	794.085	3369.599	1678.912	22255.519	10794.724		
G. avg. r		0.88		0.50		0.59		

- Version 8 has fewer failures.
- Version 8 has more firsts.
- Version 8 is at least 40% faster on average for medium to large problems.

Semidefinite problems

	small		medium		large		fails'	
	7	8	7	8	7	8	7	8
Num.	111	111	55	55	22	22	62	8
Firsts	35	94	0	55	0	22		
Total time	228.043	124.442	2484.902	1621.460	7077.588	4111.834		
G. avg. r		0.73		0.65		0.59		

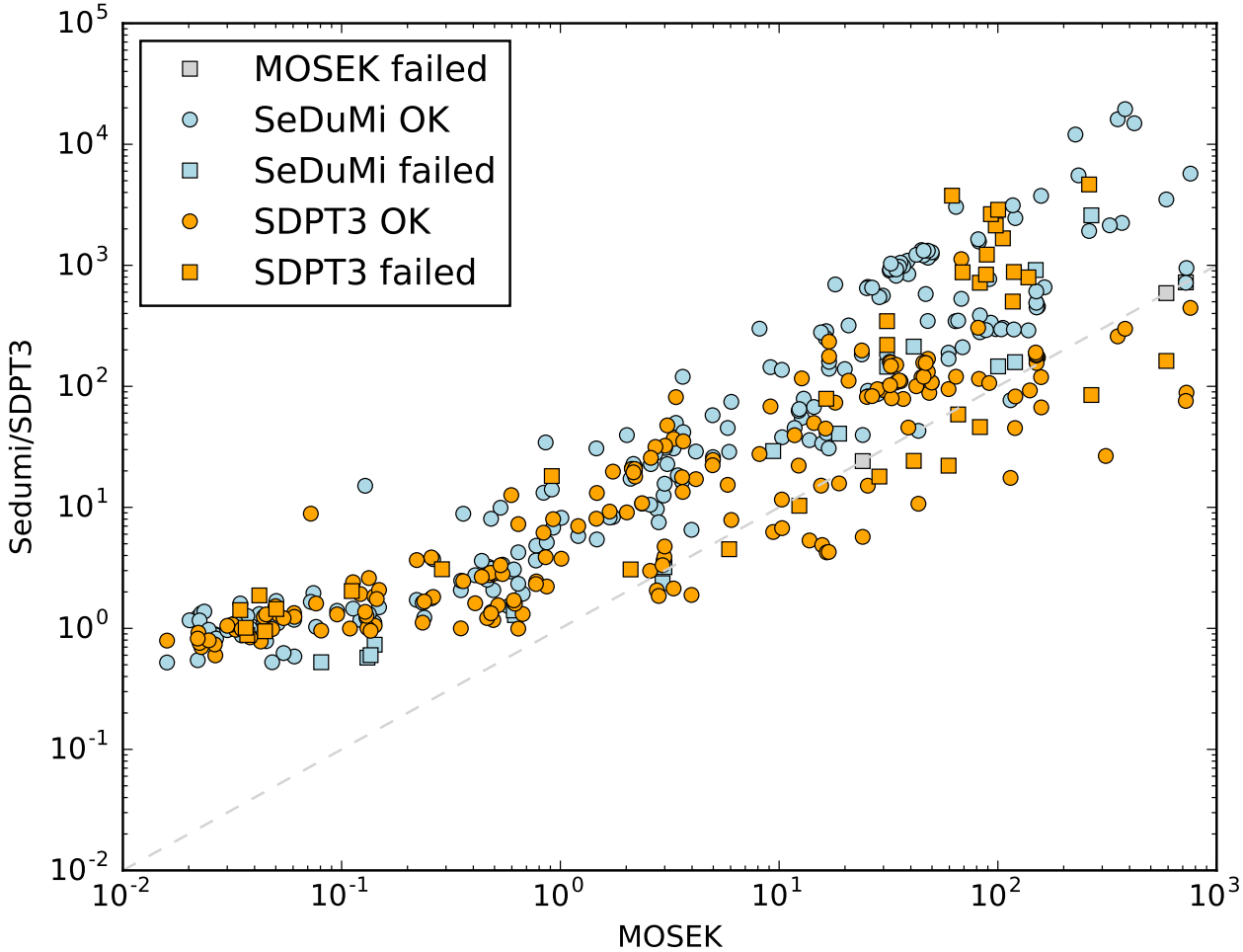
- Version 8 is more stable and has the most firsts.
- Version 8 is at least 40% faster on average for medium to large problems.

Semidefinite problems
Comparison with SeDuMi and SDPT3

- We use a subset of Mittelmann's benchmark-set + customer provided problems.
- MOSEK on 4 threads, MATLAB up to 20 threads.
- Problems are categorized as **failed** if
  - MOSEK returns unknown solution status.
  - SeDuMi returns `info.numerr==2`.
  - SDPT3 returns `info.termcode` $\notin [0, 1, 2]$ and `norm(info.dimacs,Inf)>1e-5`.

Semidefinite problems

Comparison with SeDuMi and SDPT3



Solution time for MOSEK vs SeDuMi/SDPT3 on 234 problems.

Semidefinite problems

Comparison with SeDuMi and SDPT3

	small			medium		
	MOSEK	SeDuMi	SDPT3	MOSEK	SeDuMi	SDPT3
Num.	127	127	127	63	63	63
Firsts	116	1	10	47	0	16
Total time	218.2	1299.5	843.6	2396.0	32709.8	5679.5

	large			fails		
	MOSEK	SeDuMi	SDPT3	MOSEK	SeDuMi	SDPT3
Num.	44	44	44	6	27	47
Firsts	31	0	13			
Total time	9083.8	119818.1	35268.2			

- MOSEK is fastest on average with fewest failures.
- SeDuMi is almost always slower than MOSEK.

Quadratic and quadratically constrained problems

	small		medium		large		fails'	
	7	8	7	8	7	8	7	8
Num.	466	466	19	19	8	8	28	34
Firsts	336	260	3	16	2	6		
Total time	44514.992	197.182	25958.112	415.943	5447.486	3167.290		
G. avg. r		0.97		0.27		0.64		

- Version 8 has a few more failures and slightly slower on average for small problems.
- Version 8 is faster for medium to large problems.

Mixed integer conic quadratic optimization

- Timeout set to 3600 seconds.
- Optimality gap set to 0.0.

Version	7	8
Num.	181	181
Firsts	30	58
Solved	123	128
Total time	46546	26162
G. avg. r		0.65

- Version 8 is significantly faster.
- Solves a few more problems to optimality.

Conclusion

- MOSEK version 8 is a significant improvement over version 7.
- Particularly the robustness of semidefinite optimizer has been improved dramatically.
- On average a 20% to 40% reduction in the solution time can be expected for quadratic and conic problems

Take home message

With MOSEK v8 SDO starts getting practible!