

On Using MOSEK to Solve Large-Scale Linear and Conic Optimization Problems

Erling D. Andersen

MOSEK ApS

INFORMS annual meeting
Minneapolis, October 6-9, 2013

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About MOSEK:

- Software package for solving optimization problems.
- Version 1 released 1999.
- Version 7 released 2013.

Problem types:

- Linear + integer variables.
- Conic quadratic + integer variables.
- Semidefinite optimization.
- Convex quadratic + integer variables.
- General convex.

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Several interfaces:

- Fusion API, Optimizer API and toolbox.
- Supports different languages and tools.

One optimization engine:

- Written C.
- Tuned for the large-scale sparse case.
- Exploit hardware features such as AVX instructions.

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Optimizers for continuous problems

Optimizer	Problem type			
	Network	Linear	Conic	Convex
Network simplex	+			
Primal simplex	+	+		
Dual simplex	+	+		
Interior-point	+	+	+	+

Simplex optimizers

- Large-scale sparse.
- Many options for pricing etc.

Interior-point

- Large-scale sparse with tuned linear algebra.
- Parallelized.
- Reliable infeasibility detection and reporting.

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Optimizers for mixed-integer problems

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Mixed integer conic

- Solves mixed-integer linear and conic quadratic problems.
- Parallelized.
- Run-to-run deterministic.
- Tuned for conic quadratic problems.
- No additional charge.

Mixed integer optimizer

- Solves mixed-integer linear and conic quadratic.
- Tuned for linear problems.

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Supported platforms and tools

Supported platforms operating systems

Windows, MAC OSX, Linux

MOSEK interfaces

AMPL, C/C++, Java, Python, Matlab, Microsoft .NET, R

Third party products

AIMMS, GAMS, Frontline Solver, CVX, Woodstock

Other interfaces

COIN OSI, Raven toolbox, Yalmip ...

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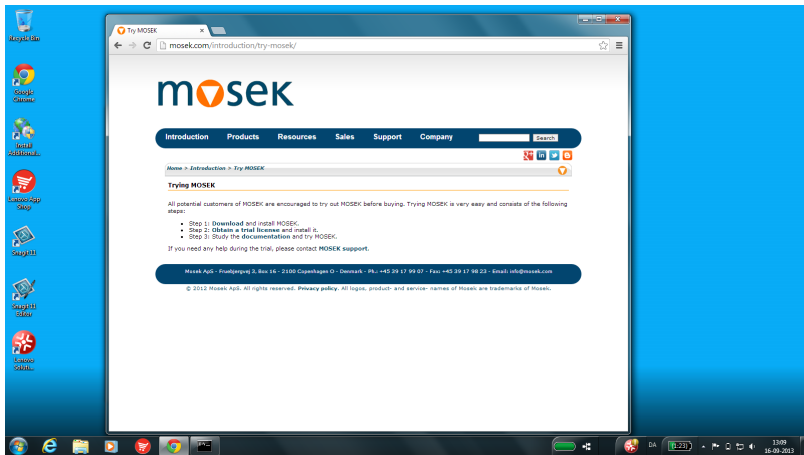
Portfolio optimization: The optimizer API

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Try MOSEK

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The screenshot shows a Windows desktop environment with a blue background. On the left side, there is a vertical taskbar with icons for MyRec Bin, Google Chrome, Intel System, Internet Explorer, Skype, Support, and Skype. At the bottom, the Windows taskbar is visible with icons for Start, Internet Explorer, Mail, VLC, and a taskbar search box. The main window is a Google Chrome browser displaying the MOSEK website. The browser's address bar shows the URL `mosek.com/introduction/try-mosek/`. The website content includes the MOSEK logo, a navigation menu with links for Introduction, Products, Resources, Sales, Support, and Company, and a search bar. Below the navigation menu, there is a breadcrumb trail: `Home > Introduction > Try MOSEK`. The main heading is **Trying MOSEK**. The text below reads: "All potential customers of MOSEK are encouraged to try out MOSEK before buying. Trying MOSEK is very easy and consists of the following steps:" followed by a list of three steps: 1. Download and install MOSEK, 2. Obtain a trial license and install it, and 3. Study the documentation and try MOSEK. Below the list, it says: "If you need any help during the trial, please contact MOSEK support." At the bottom of the page, there is contact information for Mosek ApS, including their address in Copenhagen, Denmark, phone and fax numbers, and email address. The footer also contains copyright information for 2012 Mosek ApS.

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Downloads

mosek.com/resources/download/

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Introduction Products Resources Sales Support Company

Home > Resources > Downloads

Downloads and trials

If you want to **try** or have **purchased** MOSEK, please select a download from the table below. Please note:

- You need a license e.g. a trial license or an academic license to use the software.
- The current MOSEK version is 7.0.0.93.

Click to download	Signature	Supported OS
Windows 32 bit x86	SHA512	x2 or newer
Windows 64 bit x86	SHA512	XP x64 or newer
Linux 32 bit x86	SHA512	Linux (32 bit) (glibc 2.3.4) e.g. Redhat Enterprise 5+
Linux 64 bit x86	SHA512	Linux (64 bit) (glibc 2.3.4) e.g. Redhat Enterprise 5+
MAC OS X 64 bit x86	SHA512	OSX Intel 10.7+ (64 bit)

The Windows versions require **administrator rights** to install. It is possible to use the **manual install version** which does not require administrator rights.

Frequently asked questions about the downloads

- How to install a license file?
- How to upgrade to MOSEK version 7?

Other downloads

- Version 6
- Version 5
- Version 4

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1341 16-09-2013

Obtain atrial license

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The central focus is a web browser window titled "Trial license" with the address bar showing "license.mosek.com/license2/trial/". The page content is as follows:

Request trial license for MOSEK

Please use the form below to obtain a trial license. The trial license is fully functional but limited to a period of 30 days. You are welcome to apply for an extension if the initial trial period is not enough.

The trial license is for evaluation purposes only; all commercial use is prohibited.

All fields below are mandatory!

Name:

Email:

Organization:

Below the form is a large orange downward-pointing triangle icon.

At the bottom of the page, there is a copyright notice: © 2012 Mosek Ltd. All rights reserved. [Privacy policy](#). Please, use Mosek design, brand or other logos and product and partner names of Mosek and its partners or clients.

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Markowitz portfolio optimization

- Find the optimal portfolio of assets.
- A one period model.
- Invented by H. Markowitz.
- Used extensively by hedge funds and investment companies.

The model

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Markowitz model:

$$\begin{aligned} & \text{maximize} && r^T x - \alpha s \\ & \text{subject to} && \sum_i x_i = 1 \\ & && (s, Gx) \in Q^n \\ & && x \geq 0, \end{aligned}$$

with r : average return, $G^T G$: correlation, α : risk-aversion.

- $(s, Gx) \in Q^n \iff s \geq \|Gx\|$.
- s is the std. dev. of the return i.e. risk.
- Optimize a weighted combination of return and risk.
- In practice solved for many values of α .

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A MOSEK Fusion implementation

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What is Fusion?

- An object orientated interface for building linear and conic optimization models.
- Works directly with variables and constraints.
- Easy to build and modify a model.
- Available for Java, MATLAB, .NET and Python.
- Models are similar in all languages.

Portfolio example in Python Fusion:

```
from mosek.fusion import *

def dot(x,y):
    r = 0.0
    for j in range(len(x)):
        r = r+x[j]*y[j]
    return r

def EfficientFrontier(r,G,alphas):
    n = len(r)

    M = Model('Efficient frontier')
    x = M.variable('x', n, Domain.greaterThan(0.0)) # Portfolio variables
    s = M.variable('s', 1, Domain.unbounded()) # Risk variable
    M.constraint('budget', Expr.sum(x), Domain.equalsTo(1.0)) # sum(x) = 1
    M.constraint('risk', Expr.vstack(s, Expr.mul(G,x)), Domain.inQCone()) # norm(Gx) <= s

    frontier = []
    for a in alphas:
        # objective: r'*x - a*s
        M.objective('obj', ObjectiveSense.Maximize, Expr.sub(Expr.dot(r,x), Expr.mul(a,s)))
        M.solve()
        frontier.append( (a, dot(r,x.level()), s.level()[0]) )

    return frontier

if __name__ == '__main__':

    r = [ 0.1073, 0.0737, 0.0627 ] # Vector of average returns
    G = [ [ 0.1667, 0.0232, 0.0013 ], # Cholesky Factor of Sigma.
          [ 0.0000, 0.1033, -0.0022 ],
          [ 0.0000, 0.0000, 0.0338 ] ]

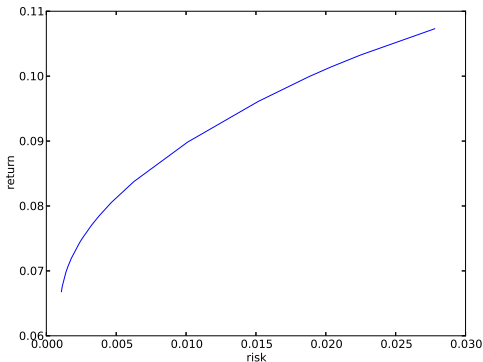
    alphas = [0.0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.75, 1.0, 1.5, 2.0, 3.0, 10.0]
    frontier = EfficientFrontier(r,DenseMatrix(G),alphas)

    print('\nEfficient frontier')
    print('%-12s %-12s %-12s' % ('alpha', 'return', 'risk'))
    for i in frontier:
        print('%-12.4f %-12.4e %-12.4e' % (i[0],i[1],i[2]))
```

Portfolio optimization

Efficient Frontier

The *efficient frontier* shows the optimal trade-off.



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Fusion summary

- Java, MATLAB and .NET Fusion looks almost identical.
- Model is close to the paper version.
- Excellent for rapid linear and conic model building.

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MATLAB toolbox implementation

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MATLAB

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- Popular among engineers, in finance, everywhere

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MOSEK optimization toolbox for MATLAB includes

- A matrix orientated interface.
- Lower level than Fusion.
- linprog, quadprog, etc clones.

MATLAB toolbox implementation

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Serializing the model

First reformulation:

$$\begin{aligned} & \text{maximize} && r^T x - \alpha s \\ & \text{subject to} && e^T x = 1 \\ & && Gx - t = 0 \\ & && (s, t) \in Q^n \\ & && x \geq 0, \end{aligned}$$

where

$$e = [1, \dots, 1]^T.$$

A new variable:

$$\bar{x} = \begin{bmatrix} x \\ t \\ s \end{bmatrix} = [x; t; s]$$

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$$\begin{aligned} & \text{maximize} && r^T x - \alpha s \\ & \text{subject to} && e^T x = 1 \\ & && Gx - t = 0 \\ & && (s, t) \in Q^n \\ & && x \geq 0, \end{aligned}$$

where

$$e = [1, \dots, 1]^T.$$

A new variable:

$$\bar{x} = \begin{bmatrix} x \\ t \\ s \end{bmatrix} = [x; t; s]$$

Serializing the model

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cont.

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Next reformulation

$$\begin{aligned}
 & \text{maximize} && \begin{bmatrix} r^T & 0_{1 \times n} & -\alpha \end{bmatrix} \bar{x} \\
 & \text{subject to} && \begin{bmatrix} e_{1 \times n} & 0_{1 \times n} & 0 \end{bmatrix} \bar{x} = 1 \\
 & && \begin{bmatrix} G & -I_{n \times n} & 0_{n \times 1} \end{bmatrix} \bar{x} = 0 \\
 & && \bar{x}_{2n+1} \geq \|\bar{x}_{(n+1):(2n)}\| \\
 & && \bar{x}_{1:n} \geq 0,
 \end{aligned}$$

MOSEK model

$$\begin{aligned}
 & \text{maximize} && c^T x \\
 & \text{subject to} && l^c \leq Ax \leq u^c \\
 & && x \in K \\
 & && l^x \leq x \leq u^x
 \end{aligned}$$

cont.

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$$\begin{aligned}
 & \text{maximize} && c^T x \\
 & \text{subject to} && l^c \leq Ax \leq u^c \\
 & && x \in K \\
 & && l^x \leq x \leq u^x
 \end{aligned}$$

Portfolio example in MATLAB:

```
[ret, res] = mosekopt('symbcon echo(0)');

r      = [ 0.1073, 0.0737, 0.0627 ]';
G      = [ [ 0.1667, 0.0232, 0.0013 ];...
          [ 0.0000, 0.1033, -0.0022 ];...
          [ 0.0000, 0.0000, 0.0338 ] ];
alphas = [0.0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.75, 1.0, 1.5, 2.0, 3.0, 10.0];
n      = length(r);
```

```
clear prob;
```

```
% The the problem.
```

```
prob.a      = [[ones(1,n),zeros(1,n),0];...
              [G,-speye(n),zeros(n,1)]];
prob.blc    = [1;zeros(n,1)];
prob.buc    = [1;zeros(n,1)];
prob.blx    = [zeros(n,1);-inf*ones(n+1,1)];
```

```
prob.cones.type = [res.symbcon.MSK_CT_QUAD];
prob.cones.sub  = [(2*n+1),(n+1):(2*n)];
prob.cones.subptr = [1];
```

```
% Compute the efficient frontier.
```

```
for i=1:length(alphas)
    alpha      = alphas(i);
    prob.c      = [r;zeros(n,1);-alpha];
    [ret,res]   = mosekopt('maximize echo(0)',prob);
    x           = res.sol.itr.xx;
    fprintf('% .2e % .4e % .4e\n',alpha,r'*x(1:n),x(2*n+1));
end
```

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MATLAB run

```
>> portfolio
```

alpha	ret	risk
0.00e+00	1.0730e-01	7.2173e-01
1.00e-01	1.0730e-01	1.6670e-01
2.00e-01	1.0730e-01	1.6670e-01
3.00e-01	8.0540e-02	6.8220e-02
4.00e-01	7.1951e-02	4.2329e-02
5.00e-01	6.9756e-02	3.7355e-02
7.50e-01	6.7660e-02	3.3827e-02
1.00e+00	6.6790e-02	3.2811e-02
1.50e+00	6.5984e-02	3.2139e-02
2.00e+00	6.5601e-02	3.1916e-02
3.00e+00	6.5221e-02	3.1758e-02
1.00e+01	6.4698e-02	3.1645e-02

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MATLAB summary

- Matrix orientated input.
- Models must be serialized.
- Can be used to replace linprog and friends.
- Similar interface is available for R.

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API model:

$$\begin{aligned} & \text{maximize} && c^T x \\ & \text{subject to} && l^c \leq Ax \leq u^c \\ & && x \in K \\ & && l^x \leq x \leq u^x \end{aligned}$$

- Serialized view. One variable only.
- Use function calls to input data.

MATLAB example thinking reuse

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Problem

$$\begin{aligned}
 & \text{maximize} && \begin{bmatrix} r^T & 0_{1 \times n} & -\alpha \end{bmatrix} \bar{x} \\
 & \text{subject to} && \begin{bmatrix} e_{1 \times n} & 0_{1 \times n} & 0 \end{bmatrix} \bar{x} = 1 \\
 & && \begin{bmatrix} G & -I_{n \times n} & 0_{n \times 1} \end{bmatrix} \bar{x} = 0 \\
 & && \bar{x}_{2n+1} \geq \| \bar{x}_{(n+1):(2n)} \| \\
 & && \bar{x}_{1:n} \geq 0,
 \end{aligned}$$

```
#include <math.h>
#include <stdio.h>

#include "mosek.h"

#define MOSEKCALL(_r,_call) ( (_r)==MSK_RES_OK ? ( (_r) = (_call) ) : ( (_r) = (_r) ) );

static void MSKAPI printstr(void *handle,
                            MSKCONST char str[])
{
    printf("%s",str);
} /* printstr */

int main(int argc, const char argv[])
{
    const MSKint32t n=3,numalpha=12;
    const double r[]={0.1073, 0.0737, 0.0627},
        G[][3]={{0.1667, 0.0232, 0.0013},
                {0.0000, 0.1033, -0.0022},
                {0.0000, 0.0000, 0.0338}},
        alphas[12]={0.0, 0.1, 0.2, 0.3, 0.4, 0.5,
                   0.75, 1.0, 1.5, 2.0, 3.0, 10.0};

    MSKenv_t      env;
    MSKint32t     k,i,j,*sub;
    MSKrescodee   res=MSK_RES_OK;
    MSKtask_t     task;

    sub = calloc(n,sizeof(MSKint32t));

    res = sub==NULL ? MSK_RES_ERR_SPACE : MSK_RES_OK;
```

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```
MOSEKCALL(res,MSK_makeenv(&env,NULL));  
MOSEKCALL(res,MSK_maketask(env,0,0,&task));  
MOSEKCALL(res,MSK_linkfunctotaskstream(task,MSK_STREAM_LOG,NULL,printstr));
```

```
/* Constraints. */
```

```
MOSEKCALL(res,MSK_appendcons(task,1+n));  
MOSEKCALL(res,MSK_putconbound(task,0,MSK_BK_FX,1.0,1.0));  
for(i=0; i<n; ++i)  
    MOSEKCALL(res,MSK_putconbound(task,1+i,MSK_BK_FX,0.0,0.0));
```

```
/* Variables. */
```

```
MOSEKCALL(res,MSK_appendvars(task,1+2*n));  
/* x variables. */  
for(j=0; j<n; ++j)  
{  
    MOSEKCALL(res,MSK_putcj(task,j,r[j]));  
    MOSEKCALL(res,MSK_putaij(task,0,j,1.0));  
    for(k=0; k<n; ++k)  
        MOSEKCALL(res,MSK_putaij(task,1+k,j,G[k][j]));  
  
    MOSEKCALL(res,MSK_putvarbound(task,j,MSK_BK_LO,0.0,MSK_INFINITY));  
}
```

```
/* t variables. */  
for(j=0; j<n; ++j)  
{  
    MOSEKCALL(res,MSK_putaij(task,1+j,n+j,-1.0));  
    MOSEKCALL(res,MSK_putvarbound(task,n+j,MSK_BK_FR,-MSK_INFINITY,MSK_INFINITY));  
}  
  
/* s variable. */  
MOSEKCALL(res,MSK_putvarbound(task,2*n,MSK_BK_FR,-MSK_INFINITY,MSK_INFINITY));  
  
sub[0] = 2*n;  
for(j=0; j<n; ++j)  
    sub[j+1] = n+j;  
MOSEKCALL(res,MSK_appendcone(task,MSK_CT_QUAD,0.0,n+1,sub));  
  
MOSEKCALL(res,MSK_putobjsense(task,MSK_OBJECTIVE_SENSE_MAXIMIZE));  
MOSEKCALL(res,MSK_putintparam(task,MSK_IPAR_LOG,0));
```

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```
for(k=0; k<numalpha; ++k)
{
    MOSEKCALL(res,MSK_putcj(task,2*n,-alphas[k]));

    /* MOSEKCALL(res,MSK_writedata(task,"dump.opf")); */

    MOSEKCALL(res,MSK_optimize(task));

    /* MSK_solutionssummary(task,MSK_STREAM_MSG); */

    if ( res==MSK_RES_OK )
    {
        double er=0.0,xj;

        for(j=0; j<n; ++j)
        {
            MOSEKCALL(res,MSK_getxxslice(task,MSK_SOL_ITR,j,j+1,&xj));
            er += r[j]*xj;
        }

        MOSEKCALL(res,MSK_getxxslice(task,MSK_SOL_ITR,2*n,2*n+1,&xj));

        printf("%e %e %e\n",alphas[k],er,xj);
    }
}

free(sub);

return ( 0 );
}
```

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Building instructions

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Using Visual studio command line tools:

```
cl portfolio.c  
  /I "\program files\mosek\7\tools\platform\win64x86\h"  
  /link "\program files\mosek\7\tools\platform\win64x86\bin\mosek64_7_0.lib"
```

on one line.

Running:

```
portfolio  
0.000000e+000 1.073000e-001 7.217338e-001  
1.000000e-001 1.073000e-001 1.667000e-001  
2.000000e-001 1.073000e-001 1.667000e-001  
3.000000e-001 8.053969e-002 6.822048e-002  
4.000000e-001 7.195059e-002 4.232918e-002  
5.000000e-001 6.975570e-002 3.735526e-002  
7.500000e-001 6.766020e-002 3.382745e-002  
1.000000e+000 6.679036e-002 3.281117e-002  
1.500000e+000 6.598434e-002 3.213941e-002  
2.000000e+000 6.560097e-002 3.191621e-002  
3.000000e+000 6.522112e-002 3.175819e-002  
1.000000e+001 6.469785e-002 3.164510e-002
```

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Debugging tips

Use writedata:

```
MSK_writedata(task, "dump.opf");
```

Content of dump.opf:

```
[objective maximize]
  1.073e-001 x0000 + 7.37e-002 x0001 + 6.270000000000001e-002 x0002 - 1e+001 x0006
[/objective]

[constraints]
[con c0000]  x0000 + x0001 + x0002 = 1e+000 [/con]
[con c0001]  1.667e-001 x0000 + 2.32e-002 x0001 + 1.3e-003 x0002 - x0003 = 0e+000 [/con]
[con c0002]  1.033e-001 x0001 - 2.2e-003 x0002 - x0004 = 0e+000 [/con]
[con c0003]  3.38e-002 x0002 - x0005 = 0e+000 [/con]
[/constraints]

[bounds]
[b]          0 <= * [/b]
[b]          x0003,x0004,x0005,x0006 free [/b]
[cone quad k0000] x0006, x0003, x0004, x0005 [/cone]
[/bounds]
```

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```
MSK_writedata(task, "dump.opf");
```

Content of dump.opf:

```
[objective maximize]
  1.073e-001 x0000 + 7.37e-002 x0001 + 6.270000000000001e-002 x0002 - 1e+001 x0006
[/objective]

[constraints]
[con c0000]  x0000 + x0001 + x0002 = 1e+000 [/con]
[con c0001]  1.667e-001 x0000 + 2.32e-002 x0001 + 1.3e-003 x0002 - x0003 = 0e+000 [/con]
[con c0002]  1.033e-001 x0001 - 2.2e-003 x0002 - x0004 = 0e+000 [/con]
[con c0003]  3.38e-002 x0002 - x0005 = 0e+000 [/con]
[/constraints]

[bounds]
[b]          0 <= * [/b]
[b]          x0003,x0004,x0005,x0006 free [/b]
[cone quad k0000] x0006, x0003, x0004, x0005 [/cone]
[/bounds]
```

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Reduce looping:

Use

```
MSK_putcslice(task,0,n,r);
```

instead of

```
for(j=0; j<n; ++j)  
    MSK_putcj(task,j,r[j]);
```

Optimizer API summary

- Harder to code against the optimizer API than the Fusion API.
- Highly efficient. Particularly for change and reoptimize.
- Optimizer API available for Java, .NET and Python too.

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MOSEK features:

- Powerful optimization engine.
- Many interfaces included.
- Extensive documentation available.
- Fusion API is easier to use than optimizer API.

Slides!

- <http://mosek.com/resources/presentations/>

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