Disjunctive conic cuts: The good, the bad, and implementation

MOSEK workshop on Mixed-integer conic optimization

Julio C. Góez
January 11, 2018

NHH Norwegian School of Economics
Motivation
Goals!

- Extend the ideas of disjunctive programming to quadratic problems.
- Derive disjunctive conic cuts for MISOCO:
  - Solve the continuous relaxation (a SOCO problem).
  - Identify a violated disjunction (fractional variable).
  - Design a cut to approximate convex hull of disjunctive set.
  - Include cuts in branch and cut algorithm.
- Show that these ideas may be used to derive valid inequalities for some non-convex quadratic sets.
Mixed integer second order cone optimization (MISOCO)

\[
\begin{align*}
\text{min: } & \quad c^T x \\
\text{s.t.: } & \quad Ax = b \\
& \quad x \in \mathbb{L}^n \\
& \quad x \in \mathbb{Z}^d \times \mathbb{R}^{n-d},
\end{align*}
\]

where:

- \( A \in \mathbb{R}^{m \times n}, \ c \in \mathbb{R}^n, \ b \in \mathbb{R}^m, \)
- \( \mathbb{L}^n = \{ x \in \mathbb{R}^n | x_1 \geq \|x_{2:n}\| \}, \)
- Rows of \( A \) are linearly independent.
- Here \( \|x\| \) denotes Euclidean norm of \( x \).
Continuous relaxation

\[
\begin{align*}
\text{min: } & \quad x_1 - 2x_2 + x_3 \\
\text{s.t.: } & \quad x_1 - 0.1x_2 + 0.2x_3 = 2.5
\end{align*}
\]

\[
x_1 \geq \left\| \begin{bmatrix} x_2 \\ x_3 \end{bmatrix} \right\|
\]

Feasible set
Find the optimal solution $x_{\text{soco}}^*$ for the continuous relaxation

$$\begin{align*}
\text{min:} & \quad 3x_1 + 2x_2 + 2x_3 + x_4 \\
\text{s.t.:} & \quad 9x_1 + x_2 + x_3 + x_4 = 10 \\
& \quad (x_1, x_2, x_3, x_4) \in \mathbb{L}^4 \\
& \quad x_4 \in \mathbb{Z}.
\end{align*}$$

Relaxing the integrality constraint we get the optimal solution:

$$x_{\text{soco}}^* = (1.36, -0.91, -0.91, -0.45),$$

with an optimal objective value: $z^* = 0.00$. 

---

**MISO**

**CO example:** Solve the relaxed problem

Find the optimal solution $x_{\text{soco}}^*$ for the continuous relaxation

$$\begin{align*}
\text{min:} & \quad 3x_1 + 2x_2 + 2x_3 + x_4 \\
\text{s.t.:} & \quad 9x_1 + x_2 + x_3 + x_4 = 10 \\
& \quad (x_1, x_2, x_3, x_4) \in \mathbb{L}^4 \\
& \quad x_4 \in \mathbb{Z}.
\end{align*}$$

Relaxing the integrality constraint we get the optimal solution:

$$x_{\text{soco}}^* = (1.36, -0.91, -0.91, -0.45),$$

with an optimal objective value: $z^* = 0.00$. 

---
MISOCO example: Reformulation

Reformulation of the relaxed problem

\[
\begin{align*}
\text{min:} & \quad \frac{1}{3} (10 + 5x_2 + 5x_3 + 2x_4) \\
\text{s.t.:} & \quad \begin{bmatrix} x_2 & x_3 & x_4 \end{bmatrix} \begin{bmatrix} 8 & -\frac{1}{10} & -\frac{1}{10} \\ -\frac{1}{10} & 8 & -\frac{1}{10} \\ -\frac{1}{10} & -\frac{1}{10} & 8 \end{bmatrix} \begin{bmatrix} x_2 \\ x_3 \\ x_4 \end{bmatrix} + 2 \begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} x_2 \\ x_3 \\ x_4 \end{bmatrix} - 10 \leq 0 \\
x_4 & \in \mathbb{Z}.
\end{align*}
\]

Feasible set of the reformulated problem
MISOCO example: Find a violated disjunction & cut

The disjunction \( x_4 \leq -1 \lor x_4 \geq 0 \) is violated by \( x^{*}_\text{soco} \).

An integer optimal solution is obtained after adding one cut:

\[
x^{*}_\text{misoco} = x^{*}_\text{soco} = (1.32, -0.93, -0.93, 0.00),
\]

with an optimal objective value: \( z^{*}_\text{misoco} = z^{*}_\text{soco} = 0.24 \).
Applications

- Computer vision and pattern recognition
  - Kumar, Torr, and Zisserman (2006).
- Portfolio optimization with round lot purchasing constraints
  - Bonami and Lejeune (2009)
- Location-inventory problems
  - Atamtürk, Berenguer, and Shen (2009)
- Joint network optimization and beamforming
  - Cheng, Drewes, Philipp, and Pesavento (2012)
- Infrastructure planning for electric vehicles
  - Mak, Rong, and Shen (2013)
- Sequencing appointments for service systems
  - Mak, Rong, and Zhang (2014)
- The design of service systems with congestion
  - Góez and Anjos (2017)
Related work

- **A Complete Characterization of Disjunctive Conic Cuts for Mixed Integer Second Order Cone Optimization**

- **Intersection cuts for nonlinear integer programming: Convexification techniques for structured sets**

- **Two-term disjunctions on the second-order cone**

- **Disjunctive cuts for cross-sections of the second-order cone**
The good
Quadratic sets and disjunctions

\[ Q = \{ x \in \mathbb{R}^n \mid x^T P x + 2p^T x + \rho \leq 0 \} \]
\[ A = \{ x \in \mathbb{R}^n \mid a^T x \geq \alpha \} \]
\[ B = \{ x \in \mathbb{R}^n \mid b^T x \leq \beta \} \]

- \( P \in \mathbb{R}^{n \times n} \) with \( n - 1 \) positive and exactly one non-positive eigenvalues, \( p \in \mathbb{R}^n \), \( \rho \in \mathbb{R} \).
- \( \|a\| = \|b\| = 1 \) and \( \beta < \alpha \).
- We denote the boundaries of the half-spaces \( A \) and \( B \) by \( A^= \) and \( B^= \) respectively.
- The intersection \( Q \cap A \cap B \) results in the disjunctive sets \( Q \cap A \) and \( Q \cap B \)
- We assume that \( A^= \cap Q \neq \emptyset \), \( B^= \cap Q \neq \emptyset \), and \( B^= \cap A^= \cap Q = \emptyset \).
A family of quadratic inequalities

Let \( \{Q(\tau) \mid \tau \in \mathbb{R}\} \) be a family of quadratic sets having the same intersection with \( A^- \) and \( B^- \), with

\[
Q(\tau) = \left\{ x \in \mathbb{R}^n \mid (x^\top P x + 2p^\top x + \rho) + \tau (a^\top x - \alpha) (b^\top x - \beta) \leq 0 \right\} \\
= \left\{ x \in \mathbb{R}^n \mid x^\top P(\tau) x + 2p(\tau)^\top x + \rho(\tau) \leq 0 \right\}.
\]

where

- \( P(\tau) = P + \tau \frac{ab^\top + ba^\top}{2} \)
- \( p(\tau) = p - \tau \frac{\beta a + \alpha b}{2} \)
- \( \rho(\tau) = \rho + \tau \alpha \beta \)
A family of quadratic inequalities

Sequence of quadrics $w^\top P(\tau)w + 2p(\tau)^\top w + \rho(\tau) \leq 0$, for $-101 \leq \tau \leq 100$. 
Family of quadrics intersecting two parallel hyperplanes

<table>
<thead>
<tr>
<th>Range</th>
<th>$\mathbf{(P(\tau), p(\tau), \rho(\tau))}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tau &gt; -8.9875$</td>
<td>Ellipsoids</td>
</tr>
<tr>
<td>$\tilde{\tau} = -8.9875$</td>
<td>Paraboloid</td>
</tr>
<tr>
<td>$-9.5903 &lt; \tau &lt; -8.9875$</td>
<td>Two sheets hyperboloids</td>
</tr>
<tr>
<td>$\tilde{\tau}_2 = -9.5903$</td>
<td>Cone</td>
</tr>
<tr>
<td>$-101.7697 &lt; \tau &lt; -9.5903$</td>
<td>One sheet hyperboloids</td>
</tr>
<tr>
<td>$\tilde{\tau}_1 = -101.7697$</td>
<td>Cone</td>
</tr>
<tr>
<td>$\tau &lt; -101.7697$</td>
<td>Two sheets hyperboloids</td>
</tr>
</tbody>
</table>

Behavior of the quadrics for different ranges of $\tau$. 
Corollary

Given a quadratic set $Q$ and two half spaces $A$ and $B$, any quadratic set in the family \( \{ Q(\tau) \mid \tau \in \mathbb{R} \} \) is a valid quadratic inequality for $Q \cap (A^= \cup B^=)$.
Theorem

Given a quadratic set $Q$ and two half spaces $A$ and $B$ such that $B^= \cap A^= \cap Q = \emptyset$, a quadratic set in the family $\{Q(\tau) \mid \tau \in \mathbb{R}\}$ is a valid quadratic inequality for $Q \cap (A \cup B)$ if and only if $\tau \leq 0$. 
A family of valid quadratic inequalities

Theorem

Consider a quadratic set $Q$ and two half-spaces $A$ and $B$. If there exists a $\bar{\tau}$ such that $Q(\bar{\tau}) = Q_1(\bar{\tau}) \cup Q_2(\bar{\tau})$ is a non-convex quadratic cone, and its vertex $v$ is contained in $A$ or $B$ but not in $A \cap B$, then each branch $i = 1, 2$ of $Q(\bar{\tau})$ is a valid quadratic inequality for $Q \cap (A_i^\infty(\bar{\tau}) \cup B_i^\infty(\bar{\tau}))^1$, such that

$$\text{conv}(Q \cap (A_i^\infty(\bar{\tau}) \cup B_i^\infty(\bar{\tau}))) = \text{conv}(Q_i(\bar{\tau}) \cap (A_i^\infty(\bar{\tau}) \cup B_i^\infty(\bar{\tau}))) \subseteq Q_i(\bar{\tau}).$$

$^1A_i^\infty(\bar{\tau}) = A^\infty \cap Q_i(\bar{\tau})$, $A_i^\infty(\bar{\tau}) = A^\infty \cap Q_i(\bar{\tau})$, and similarly we define $B_i^\infty(\bar{\tau})$, $B_i^\infty(\bar{\tau})$. 

Parallel hyperplanes

Let \( \{ Q(\tau) \mid \tau \in \mathbb{R} \} \) be a family of quadratic sets having the same intersection with \( A^- \) and \( B^- \), with

\[
Q(\tau) = \{ x \in \mathbb{R}^n \mid (x^T P x + 2p^T x + \rho) + \tau (a^T x - \alpha) (a^T x - \beta) \leq 0 \}
= \{ x \in \mathbb{R}^n \mid x^T P(\tau) x + 2p(\tau)^T x + \rho(\tau) \leq 0 \}
\]

where

- \( P(\tau) = P + \tau aa^T \)
- \( p(\tau) = p - \tau \frac{\beta + \alpha}{2} a \)
- \( \rho(\tau) = \rho + \tau \alpha \beta \)
Non-convex quadratic cones in the family

Rewrite \( Q(\tau) \) as:

\[
Q(\tau) = \{ x \in \mathbb{R}^n \mid (x + P^{-1}(\tau)p(\tau)) P(\tau) (x + P^{-1}(\tau)p(\tau)) \leq p(\tau)P^{-1}(\tau)p(\tau) - \rho(\tau) \}
\]

and we obtain

\[
p(\tau)^\top P(\tau)^{-1}p(\tau) - \rho(\tau) = \frac{(1 - 2a_1^2) \frac{(\alpha - \beta)^2}{4} \tau^2 - (\rho(1 - 2a_1^2) + \alpha\beta)\tau - \rho}{1 + \tau(1 - 2a_1^2)}.
\]

The roots \( \bar{\tau}_1 \leq \bar{\tau}_2 \) of the numerator are

\[
2 \left( \frac{\rho(1 - 2a_1^2) + \alpha\beta \pm \sqrt{(\rho(1 - 2a_1^2) + \beta^2)(\rho(1 - 2a_1^2) + \alpha^2)}}{(1 - 2a_1^2)(\alpha - \beta)^2} \right)
\]
Lemma

Let $Q = Q_1 \cup Q_2$ be one of the quadratic sets in our list. We have the following cases:

- If $a_1^2 > \frac{1}{2}$ and the set $Q(\bar{\tau}_2)$ is a non-convex quadratic cone, then its vertex $v$ is either in $A$ or $B$.
- If $a_1^2 = \frac{1}{2}$ and $\alpha \beta \geq 0$, then the set $Q(-\frac{\rho}{\alpha \beta})$ is a non-convex quadratic cone with its vertex $v$ is either in $A$ or $B$.
- If $a_1^2 < \frac{1}{2}$ and the set $Q(\bar{\tau}_1)$ is a non-convex quadratic cone, then its vertex $v$ is either in $A$ or $B$. 
Intersection of an affine space and a second order cone

- Null space representation of the affine space $\mathcal{H} = \{x \in \mathbb{R}^n \mid Ax = b\}$

  $$\mathcal{H} := \{x \in \mathbb{R}^n \mid x = x^0 + Hw, \ \forall w \in \mathbb{R}^\ell\},$$

  where $\ell = n - m$, $Ax^0 = b$, and $H \in \mathbb{R}^{n \times \ell}$ is a basis for $\text{Null}(A)$.

- There exist a matrix $P \in \mathbb{R}^{\ell \times \ell}$, $p \in \mathbb{R}^\ell$, $\rho \in \mathbb{R}$, s.t.

  $$\mathcal{F} = \mathcal{H} \cap \mathbb{L}^n = \{x \in \mathbb{R}^n \mid \exists w \in \mathcal{F}^Q, x = x^0 + Hw\},$$

  where

  $$\mathcal{F}^Q = \{w \in \mathbb{R}^\ell \mid w^T Pw + 2p^T w + \rho \leq 0, \ x_1^0 + H_1^T w \geq 0\}$$
Intersection of an affine space and a second order cone

Theorem

The matrix $P$ in the definition of the quadratic set $\mathcal{F}^Q$ has at most one non-positive eigenvalue, and at least $\ell - 1$ positive eigenvalues.

We only need to account for the following possible shapes for $\mathcal{F}^Q$:

- Ellipsoid.
- Paraboloid.
Intersection of an affine space and a second order cone

**Theorem**

The matrix $P$ in the definition of the quadratic set $\mathcal{F}^Q$ has at most one non-positive eigenvalue, and at least $\ell - 1$ positive eigenvalues.

We only need to account for the following possible shapes for $\mathcal{F}^Q$:

One branch of a hyperboloid.  
Second order cone.
Let $\mathcal{A} = \{ w \in \mathbb{R}^l \mid a^\top w \geq \alpha \}$ and $\mathcal{B} = \{ w \in \mathbb{R}^l \mid a^\top w \leq \beta \}$. Define $\mathcal{F}^D = \{ x \in \mathbb{R}^n \mid \exists w \in \mathcal{F}^Q \cap (\mathcal{A} \cup \mathcal{B}), x = x^0 + Hw \}$

**Lemma**

*Given a vector $\hat{x} \in \mathcal{F}$ and a vector $\hat{w} \in \mathcal{F}^Q$ such that $\hat{x} = x^0 + H\hat{w}$. Then $\hat{x} \notin \mathcal{F}^D$ if and only if $\hat{w} \notin \mathcal{F}^Q \cap (\mathcal{A} \cup \mathcal{B})$.***

**Proof.**

Note that any $x \in \mathcal{F}$ is a linear combination of $x^0$ and the columns of $H$. Additionally, recall that the columns of $H$ are linearly independent. Then, the vector $\hat{w}$ defining $\hat{x}$ is unique. The result follows. \qed
Disjunctive conic cuts: General theory

- Study the intersection of a convex set $\mathcal{E}$ and a disjunctive set $\mathcal{A} = \{ x \in \mathbb{R}^n \mid a^T x \geq \alpha \} \cup \mathcal{B} = \{ x \in \mathbb{R}^n \mid b^T x \leq \beta \}$.
- Show that under some mild assumptions $\text{conv} (\mathcal{E} \cap (\mathcal{A} \cup \mathcal{B}))$ can be characterized using a convex cone $\mathcal{K}$.

\[ \mathcal{A} = \{ x \in \mathbb{R}^n \mid a^T x = \alpha \} \quad \text{and} \quad \mathcal{B} = \{ x \in \mathbb{R}^n \mid b^T x = \beta \} \]
Definition
A closed convex cone $\mathcal{K} \in \mathbb{R}^n$ with $\dim(\mathcal{K}) > 1$ is called a **Disjunctive Conic Cut** (DCC) for $\mathcal{E}$ and the disjunctive set $\mathcal{A} \cup \mathcal{B}$ if

$$\text{conv}(\mathcal{E} \cap (\mathcal{A} \cup \mathcal{B})) = \mathcal{E} \cap \mathcal{K}.$$ 

Assumption

The intersection $\mathcal{A} \cap \mathcal{B} \cap \mathcal{E}$ is empty.

Assumption

The intersections $\mathcal{E} \cap \mathcal{A}^-$ and $\mathcal{E} \cap \mathcal{B}^-$ are nonempty and bounded.
Proposition

A closed convex cone $\mathcal{K} \in \mathbb{R}^n$ with $\dim(\mathcal{K}) > 1$ is a DCC for $\mathcal{E}$ and the disjunctive set $\mathcal{A} \cup \mathcal{B}$, if

$$\mathcal{K} \cap \mathcal{A}^\perp = \mathcal{E} \cap \mathcal{A}^\perp \quad \text{and} \quad \mathcal{K} \cap \mathcal{B}^\perp = \mathcal{E} \cap \mathcal{B}^\perp.$$
DCCs for MISOCO when intersections are bounded

**Theorem**

Let $A = \{ w \in \mathbb{R}^\ell | a^\top w = \alpha \}$ and $B = \{ w \in \mathbb{R}^\ell | a^\top w = \beta \}$ be given. If the sets $A \cap F^Q$ and $B \cap F^Q$ are bounded, then the quadric $Q(\bar{\tau}_2)$ contains a DCC for MISOCO.
DCCs for MISOCO when intersections are unbounded

Theorem

Let \( A^= = \{ w \in \mathbb{R}^\ell | a^T w = \alpha \} \) and \( B^= = \{ w \in \mathbb{R}^\ell | a^T w = \beta \} \) be given.

If \( P \) is non-singular and the sets \( A^= \cap F^Q \) and \( B^= \cap F^Q \) are unbounded, then the quadric \( Q(\bar{\tau}_1) \) contains a DCC for MISOCO.
DCCs for MISOCO when intersections are unbounded

**Theorem**

Let $\mathcal{A}^\perp = \{ w \in \mathbb{R}^\ell \mid a^\top w = \alpha \}$ and $\mathcal{B}^\perp = \{ w \in \mathbb{R}^\ell \mid a^\top w = \beta \}$ be given. If $P$ is **singular** and the sets $\mathcal{A}^\perp \cap \mathcal{F}^Q$ and $\mathcal{B}^\perp \cap \mathcal{F}^Q$ are **unbounded**, then the quadric $Q(\hat{\tau})$ is a DCC for MISOCO.
What happens if the hyperplanes are non-parallel?

The results for the bounded intersections still hold.
What happens if the hyperplanes are non-parallel?

The results for the bounded intersections still hold.
Does this approach work beyond MISOCO?

Let us consider

- Hyperboloids of two sheets and non-convex quadratic cones.
- Hyperboloids of one sheet.
The sets $Q \cap A^{=} \text{ and } Q \cap B^{=} \text{ are bounded, } a_1 > \frac{1}{2}$

Valid conic inequality $Q(\bar{\tau}_2)$ when both hyperplanes intersecting the same branch of $Q$
The sets $Q \cap A^= \text{ and } Q \cap B^=$ are bounded, $a_1 > \frac{1}{2}$
The sets $Q \cap A^\neq$ and $Q \cap B^\neq$ are unbounded, $a_1 = \frac{1}{2}$

Valid conic inequality $Q(\overline{\tau}_2)$, in this case $\text{conv}(Q_1 \cap (A \cup B)) = Q_1 \cap Q(\overline{\tau})$
The sets $Q \cap A^-$ and $Q \cap B^-$ are unbounded, $a_1 < \frac{1}{2}$.

Valid conic inequality $Q(\bar{\tau}_1)$, \( \text{conv}(Q_1 \cap (A \cup B)) = Q_1 \cap Q(\tau_1) \) and \( \text{conv}(Q_2 \cap (A \cup B)) = Q_2 \cap Q(\tau_1) \)
The sets $Q \cap A^\infty$ and $Q \cap B^\infty$ are bounded, $a_1 > \frac{1}{2}$.
The sets $Q \cap A^= \text{ and } Q \cap B^=$ are unbounded

\[ \beta^2 \geq 1 - 2a_1^2 \text{ and } \alpha^2 \geq 1 - 2a_1^2 \]

Valid conic inequality $Q(\bar{\tau}_1), \alpha \beta > 0$

Valid conic inequality $Q(\bar{\tau}_1), \alpha \beta < 0$
The sets $Q \cap A^\infty$ and $Q \cap B^\infty$ are unbounded

$\beta^2 \leq 1 - 2a_1^2$ and $\alpha^2 \leq 1 - 2a_1^2$

$Q(\bar{\tau}_1)$ is a cylinder defined by a hyperboloid of one sheet
• We provided valid inequalities for the cross-sections of a non-convex quadratic cone.
• Showed that these valid inequalities consider the DCCs for MISOCO.
• Investigate the potential to use the family of quadrics with some other quadratic sets.
The bad
Definition (Shahabsafa, G., Terlaky)
Let $\mathcal{X} \in \mathbb{R}^n$ be a closed convex set, and consider the disjunction $A \cup B$. If $\text{conv}(\mathcal{X} \cap (A \cup B)) = \mathcal{X}$, then disjunction $A \cup B$ is pathological for the set $\mathcal{X}$. 
Corollary (Shahabsafa, G., Terlaky)

If the following two conditions are satisfied for the set \( \hat{\mathcal{Q}} \) defined, and the disjunctive set, then we have a redundant DCC:

1. The matrix \( P \) has exactly \( n - 1 \) positive eigenvalues and one negative eigenvalue, and \( p^TP^{-1}p - \rho = 0 \);
2. The vertex of the cone \( v = P^{-1}p \) satisfies either \( \hat{a}^T v \geq \hat{\beta} \), or \( \hat{a}^T v \leq \hat{\alpha} \).
Identification of a redundant DCC for MISOCO

Hyperboloid intersection
(Redundant DCC)

Hyperboloid intersection and the DCC (not a redundant DCC)
Identification of a redundant DCC for MISOCO

Ellipsoid intersection (Redundant DCC)

Paraboloid intersection (Redundant DCC)
Corollary (Shahabsafa, G., Terlaky)

Consider the set $\hat{Q}$, as defined, and a disjunction. We have a cylindrical redundant DCyC if the following two conditions are satisfied:

1. System $\left[ \begin{array}{c} P \\ p \end{array} \right]^T d = 0$, for $d \neq 0$, has a solution.
2. System $\left[ \begin{array}{c} P \\ p \end{array} \right] y = \hat{a}$, for $y \in \mathbb{R}^{\ell+1}$, does not have a solution.
Identification of a redundant DCyC for MISOCO

A cylindrical redundant DCyC

Not a cylindrical redundant DCyC
Conclusions

• We presented two fundamental pathological cases, which help to identify when a DCC is redundant.

• The identification of the pathological cases is important for an efficient implementation of DCCs or derivation of them.

• The identification of the pathological cases of DCCs for MISOCO highlights both the limitations and the opportunities for the efficient implementation of the DCCs.
Implementation
Implementation challenges

- Generating DCC may messes up the structure of the problem, the matrices associated with the cuts are usually dense.
- DCC generation brings numerical challenges.
- Adding DCC may increase the solution time of the linear relaxations.
- No efficient warm start is available for interior point methods.
- Quadratic constraints corresponding to Euclidean-distance

\[(x_1 - 17.5)^2 + (x_5 - 7)^2 + 6814 \times b_{33} \leq 6850.\]

- All integer variables are binary, for example in the illustrative constraint the binary variable is \(b_{33}\).

<table>
<thead>
<tr>
<th></th>
<th>0203M</th>
<th>0204M</th>
<th>0205M</th>
<th>0303M</th>
<th>0304M</th>
<th>0305M</th>
</tr>
</thead>
<tbody>
<tr>
<td>Var</td>
<td>31</td>
<td>52</td>
<td>81</td>
<td>34</td>
<td>57</td>
<td>86</td>
</tr>
<tr>
<td>Binary</td>
<td>18</td>
<td>21</td>
<td>50</td>
<td>21</td>
<td>36</td>
<td>55</td>
</tr>
<tr>
<td>Constraints</td>
<td>55</td>
<td>91</td>
<td>136</td>
<td>67</td>
<td>107</td>
<td>156</td>
</tr>
<tr>
<td>Quad</td>
<td>24</td>
<td>32</td>
<td>40</td>
<td>36</td>
<td>48</td>
<td>60</td>
</tr>
</tbody>
</table>
Constrained layout problems, Bonami et al. 2008

CLay Quadratic Constraints

DCC cut
Constrained layout problems, Bonami et al. 2008

This could be done in the preprocessing phase

CLay Quadratic Constraints

DCC cut
• OsiConic: A generic interface class for SOCP solvers. This interface provides a way to build and solve SOCPs that is uniform across a variety of solvers, as well as a standard interface for querying the results.

• OsiXxxxx: Implementations of the interface for various open source and commercial solvers.

• COLA: A solver for SOCP that implements the cutting-plane Algorithm.

• CglConic: A library of procedures for generating valid inequalities for MISOCOP.

• DisCO: A solver library for MISOCOP that uses all the libraries mentioned. This library implements classical branch-and-bound type of algorithm and and outer approximation branch-and-cut algorithm.
CglConic, A Cut Library for MISCOP

Linear Case

- CglCutGenerator
- CglMixedIntegerRounding

Conic Case

- CglConicCutGenerator
- CglConicGD1
- CglConicMIR
## COLA statistics on Góez’s random instances

<table>
<thead>
<tr>
<th>instance</th>
<th>NC</th>
<th>LC</th>
<th>NUMLP</th>
<th>CPU</th>
</tr>
</thead>
<tbody>
<tr>
<td>r12c15k5i10</td>
<td>5</td>
<td>3</td>
<td>5</td>
<td>0.01</td>
</tr>
<tr>
<td>r14c18k3i9</td>
<td>3</td>
<td>6</td>
<td>16</td>
<td>0.01</td>
</tr>
<tr>
<td>r17c30k3i12</td>
<td>3</td>
<td>10</td>
<td>74</td>
<td>0.07</td>
</tr>
<tr>
<td>r17c20k5i15</td>
<td>5</td>
<td>4</td>
<td>4</td>
<td>0.0</td>
</tr>
<tr>
<td>r22c30k10i20</td>
<td>10</td>
<td>3</td>
<td>8</td>
<td>0.02</td>
</tr>
<tr>
<td>r22c40k10i20</td>
<td>10</td>
<td>4</td>
<td>22</td>
<td>0.03</td>
</tr>
<tr>
<td>r23c45k3i21</td>
<td>3</td>
<td>15</td>
<td>148</td>
<td>0.25</td>
</tr>
<tr>
<td>r27c50k5i25</td>
<td>5</td>
<td>10</td>
<td>77</td>
<td>0.11</td>
</tr>
<tr>
<td>r32c45k15i30</td>
<td>15</td>
<td>3</td>
<td>6</td>
<td>0.0</td>
</tr>
<tr>
<td>r32c60k15i30</td>
<td>15</td>
<td>4</td>
<td>32</td>
<td>0.02</td>
</tr>
<tr>
<td>r52c75k5i35</td>
<td>5</td>
<td>15</td>
<td>74</td>
<td>0.15</td>
</tr>
</tbody>
</table>
Performance Profile of CPU Time using bb-lp with disjunctive cuts

CPU Time in seconds

- disco-oa-dc-all
- disco-oa-dc-best
- disco-oa-nomilpcuts
Performance Profile of Number of Nodes Processed using bb-lp with disjunctive cuts

![Graph showing performance profile of number of nodes processed using bb-lp with disjunctive cuts. The graph compares three methods: disco-oa-dc-all, disco-oa-dc-best, and disco-oa-nomilpcuts. The x-axis represents the number of nodes, and the y-axis shows the profiling range from 0 to 0.5. Each method is represented by a different line color.](image-url)
Conclusions and future work
Conclusions and future work

• We provided an extension of disjunctive programming to MISOCO problems.
• We were able to provide closed forms for the derivation of DCCs for MISOCO problems.
• This work gives a full characterization of DCCs for MISOCO problems when using parallel disjunctions.
• We provided valid inequalities for the cross-sections of a non-convex quadratic cone and a one sheet hyperboloid.
• Investigate the potential to use the family of quadrics with some other quadratic sets.
• Investigate the computational potential of this inequalities.


Tusen Takk!!!