Mixed-integer conic optimization using MOSEK

January 11th 2018

joachim.dahl@mosek.com

www.mosek.com

(Extremely) disciplined convex programming

- Proving (or disproving) convexity of a general nonlinear optimization problem is difficult.
- Disciplined convex programming (DCP) (Grant & Boyd) gives a rule-set for combining convex operators that maintain convexity. Models are mostly converted to symmetric conic form.
- Lubin [1] shows all convex instances (85) in the MINLPLIB2 library are conic representable using only 5 cones (some of them nonsymmetric).
- Using the nonsymmetric conic formulation leads to *extremely disciplined convex programming*. Simple, yet flexible for modeling, with efficient numerical algorithms.

Conic optimization

Linear cone problem:

$$\begin{array}{ll} \text{minimize} & c^T x \\ \text{subject to} & Ax = b \\ & x \in \mathcal{K}, \end{array}$$

with $\mathcal{K} = \mathcal{K}_1 \times \mathcal{K}_2 \times \cdots \times \mathcal{K}_K$ a product of proper cones.

Dual:

$$\begin{array}{ll} \text{maximize} & b^T y \\ \text{subject to} & c - A^T y = s \\ & s \in \mathcal{K}^*, \end{array}$$

with $\mathcal{K}^* = \mathcal{K}_1^* \times \mathcal{K}_2^* \times \cdots \times \mathcal{K}_K^*$.

Symmetric cones (supported by MOSEK 8)

- *the nonnegative orthant*

$$\mathcal{K}_l^n := \{x \in \mathbb{R}^n \mid x_j \geq 0,\ j = 1, \ldots, n\},$$

- *the quadratic cone*

$$\mathcal{K}_q^n = \{x \in \mathbb{R}^n \mid x_1 \geq \left(x_2^2 + \cdots + x_n^2\right)^{1/2}\},$$

- *the rotated quadratic cone*

$$\mathcal{K}_r^n = \{x \in \mathbb{R}^n \mid 2x_1x_2 \geq x_3^2 + \ldots x_n^2,\ x_1, x_2 \geq 0\}.$$

- *the semidefinite matrix cone*

$$\mathcal{K}_s^n = \{x \in \mathbb{R}^{n(n+1)/2} \mid z^T \mathbf{mat}(x) z \geq 0,\ \forall z\},$$

$$\text{with } \mathbf{mat}(x) := \begin{bmatrix} x_1 & x_2/\sqrt{2} & \ldots & x_n/\sqrt{2} \\ x_2/\sqrt{2} & x_{n+1} & \ldots & x_{2n-1}/\sqrt{2} \\ \vdots & \vdots & & \vdots \\ x_n/\sqrt{2} & x_{2n-1}/\sqrt{2} & \ldots & x_{n(n+1)/2} \end{bmatrix}.$$

Nonsymmetric cones (in next MOSEK release)

- *the three-dimensional power cone*

$$\mathcal{K}_p^\alpha = \{x \in \mathbb{R}^3 \mid x_1^\alpha x_2^{(1-\alpha)} \geq |x_3|,\ x_1, x_2 > 0\},$$

  for $0 < \alpha < 1$.
- *the exponential cone*

$$\mathcal{K}_e = \mathrm{cl}\{x \in \mathbb{R}^3 \mid x_1 \geq x_2 \exp(x_3/x_2),\ x_2 > 0\}.$$

IPMs for nonsymmetric cones are less studied, and less mature.

Examples of quadratic cones

- Absolute value:

$$|x| \leq t \quad \Longleftrightarrow \quad (t, x) \in \mathcal{K}_q^2.$$

- Euclidean norm:

$$\|x\|_2 \leq t \quad \Longleftrightarrow \quad (t, x) \in \mathcal{K}_q^{n-1},$$

- Second-order cone inequality:

$$\|Ax + b\|_2 \leq c^T x + d \quad \Longleftrightarrow \quad (c^T x + d, Ax + b) \in \mathcal{K}_q^{m+1}.$$

Examples of rotated quadratic cones

- Squared Euclidean norm:

$$\|x\|_2^2 \leq t \quad \iff \quad (1/2, t, x) \in \mathcal{K}_r^{n+2}.$$

- Convex quadratic inequality:

$$(1/2)x^T Q x \leq c^T x + d \quad \iff \quad (1/2, c^T x + d, F^T x) \in \mathcal{K}_r^{k+2}$$

with $Q = F^T F$, $F \in \mathbb{R}^{n \times k}$.

Examples of rotated quadratic cones

- Convex hyperbolic function:

$$\frac{1}{x} \leq t,\ x > 0 \quad \Longleftrightarrow \quad (x, t, \sqrt{2}) \in \mathcal{K}_r^3.$$

- Convex negative rational power:

$$\frac{1}{x^2} \leq t,\ x > 0 \quad \Longleftrightarrow \quad (t, \frac{1}{2}, s), (x, s, \sqrt{2}) \in \mathcal{K}_r^3.$$

- Square roots:

$$\sqrt{x} \geq t,\ x \geq 0 \quad \Longleftrightarrow \quad (\frac{1}{2}, x, t) \in \mathcal{K}_r^3.$$

- Convex positive rational power:

$$x^{3/2} \leq t,\ x \geq 0 \quad \Longleftrightarrow \quad (s, t, x), (x, 1/8, s) \in \mathcal{K}_r^3.$$

Examples of power cones

- Models many quadratic cone examples more succinctly.
- Powers:

$$t \geq |x|^p \quad \iff \quad (t, 1, x) \in \mathcal{K}_p^{1/p}$$

- p-norm cones ($p > 1$):

$$t \geq \|x\|_p \quad \iff \quad \sum r_i = t,\ (r_i, t, x_i) \in \mathcal{K}_p^{1/p},\ i = 1, \ldots, n.$$

Examples of exponential cones

- Expontial:

$$e^x \leq t \quad \Longleftrightarrow \quad (t, 1, x) \in \mathcal{K}_e.$$

- Logarithm:

$$\log x \geq t \quad \Longleftrightarrow \quad (x, 1, t) \in \mathcal{K}_e.$$

- Entropy:

$$-x \log x \geq t \quad \Longleftrightarrow \quad (1, x, t) \in \mathcal{K}_e.$$

- Softplus function:

$$\log(1+e^x) \leq t \quad \Longleftrightarrow \quad (u, 1, x-t), (v, 1, -t) \in \mathcal{K}_e,\ u+v \leq 1.$$

- Log-sum-exp:

$$\log(\sum_i e^{x_i}) \leq t \Longleftrightarrow \sum u_i \leq 1,\ (u_i, 1, x_i-t) \in \mathcal{K}_e,\ i = 1, \ldots, n.$$

The homogeneous model for conic problems

Solution to the homogenous model

$$
\begin{aligned}
Ax - b\tau &= 0\\
c\tau - A^T y - s &= 0\\
c^T x - b^T y + \kappa &= 0\\
x \in \mathcal{K},\ s \in \mathcal{K}^*,\ \tau, \kappa &\geq 0,
\end{aligned}
$$

encapsulates different duality cases:

- If $\tau > 0$, $\kappa = 0$ then $\frac{1}{\tau}(x, y, s)$ is optimal,

$$Ax = b\tau, \quad c\tau - A^T y = s, \quad c^T x - b^T y = 0.$$

- If $\tau = 0$, $\kappa > 0$ then the problem is infeasible,

$$Ax = 0, \quad -A^T y = s, \quad c^T x - b^T y < 0.$$

- If $\tau = 0$, $\kappa = 0$ then the problem is ill-posed.

Shifted central-path for cone problems

Central-path for interior point $(x^0, s^0, y^0, \tau^0, \kappa^0)$:

$$
\begin{gathered}
Ax_\mu - b\tau_\mu = \mu(Ax^0 - b\tau^0) \\
s_\mu + A^T y_\mu - c\tau_\mu = \mu(s^0 + A^T y^0 - c\tau^0) \\
c^T x_\mu - b^T y_\mu + \kappa_\mu = \mu(c^T x^0 - b^T y^0 + \kappa^0) \\
s_\mu = -\mu F'(x_\mu), \quad x_\mu = -\mu F_*'(s_\mu), \quad \kappa_\mu \tau_\mu = \mu,
\end{gathered}
$$

parametrized by μ. Equivalently we have

$$
\begin{gathered}
\begin{bmatrix} 0 & A & -b \\ -A^T & 0 & c \\ b^T & -c^T & 0 \end{bmatrix} \begin{bmatrix} y_\mu \\ x_\mu \\ \tau_\mu \end{bmatrix} - \begin{bmatrix} 0 \\ s_\mu \\ \kappa_\mu \end{bmatrix} = \mu \begin{bmatrix} r_p^0 \\ r_d^0 \\ r_g^0 \end{bmatrix} \\
s_\mu = -\mu F'(x_\mu), \quad x_\mu = -\mu F_*'(s_\mu), \quad \kappa_\mu \tau_\mu = \mu,
\end{gathered}
$$

where

$$
r_p^0 = Ax^0 - b, \qquad r_d^0 = c\tau^0 - A^T y^0 - s^0, \qquad r_g^0 = b^T y^0 - c^T x^0 - \kappa^0.
$$

Symmetric cones

Symmetric cones are a convex cones with a bilinear product:

1. $u \circ v = v \circ u$,
2. $u \circ (u^2 \circ v) = u^2 \circ (u \circ v)$,
3. $u^T(v \circ w) = (u \circ v)^T w$.

Defines roots, inverses, etc.,

$$u^{1/2} \circ u^{1/2} = u, \quad u^{-1} \circ u = u \circ u^{-1} = e.$$

- Nonnegative orthant: $u \circ v = \mathbf{diag}(u)v$, $e = (1, 1, \ldots, 1)$.
- Second-order cone:

$$u \circ v = \begin{bmatrix} u^T v \\ u_1 v_{2:n} + v_1 u_{2:n} \end{bmatrix}, \; e = (1, 0, \ldots, 0)$$

- Semidefinite cone:

$$u \circ v = (1/2)\mathbf{vec}(\mathbf{mat}(u)\mathbf{mat}(v) + \mathbf{mat}(v)\mathbf{mat}(u)), e = \mathbf{vec}(I).$$

Symmetric cones

For the symmetric cones, we have

$$F'(x) = -x^{-1}$$

(using the inverse defined by the product), so the centrality condition becomes

$$s = -\mu F'(x) = \mu x^{-1},$$

or equivalently

$$x \circ s = \mu e.$$

Symmetric cones, Nesterov-Todd scaling

Properties of Nesterov-Todd scaling W:

- maps x and s to the same *scaling point* λ,

$$\lambda = Wx = W^{-1}s,$$

- leaves the cone invariant,

$$x, s \succeq_{\mathcal{K}} 0 \quad \Longleftrightarrow \quad \lambda \succeq_{\mathcal{K}} 0,$$

- preserves the central path,

$$x \circ s = \mu e \quad \Longleftrightarrow \quad \lambda \circ \lambda = \mu e.$$

Linearized scaled centrality:

$$\lambda \circ (W\Delta x + W^{-1}\Delta s) = \mu e - \lambda^2.$$

Scaling for nonsymmetric cones

Following Tunçel [4] we consider a scaling $W^T W \succ 0$,

$$\lambda = Wx = W^{-T}s, \qquad \tilde{\lambda} = W\tilde{x} = W^{-T}\tilde{s}$$

where $\tilde{x} := -F_*'(s)$ and $\tilde{s} := -F'(x)$. The centrality conditions

$$x = \mu\tilde{x}, \quad s = \mu\tilde{s}$$

can then be written symmetrically as

$$\lambda = \mu\tilde{\lambda},$$

and we linearize the centrality condition $v = \mu\tilde{v}$ as

$$W\Delta x + W^{-T}\Delta s = -\lambda + \mu\tilde{\lambda}.$$

Search-direction for nonsymmetric cones

Search-direction with centering parameter γ:

$$\begin{bmatrix} 0 & A & -b \\ -A^T & 0 & c \\ b^T & -c^T & 0 \end{bmatrix} \begin{bmatrix} \Delta y \\ \Delta x \\ \Delta \tau \end{bmatrix} - \begin{bmatrix} 0 \\ \Delta s \\ \Delta \kappa \end{bmatrix} = (\gamma - 1) \begin{bmatrix} r_p^0 \\ r_d^0 \\ r_g^0 \end{bmatrix}$$

$$W \Delta x + W^{-1} \Delta s = \gamma \mu \tilde{\lambda} - \lambda, \quad \tau \Delta \kappa + \kappa \Delta \tau = \gamma \mu - \tau \kappa.$$

Constant decrease of residuals:

$$\begin{aligned} A(x + \alpha \Delta x) &= (1 - \alpha(1 - \gamma)) r_p^0 \\ c(\tau + \alpha \Delta \tau) - A^T(y + \alpha \Delta y) - (s + \alpha \Delta s) &= (1 - \alpha(1 - \gamma)) r_d^0 \\ b^T(y + \alpha \Delta y) - c^T(x + \alpha \Delta x) - (\kappa + \alpha \Delta \kappa) &= (1 - \alpha(1 - \gamma)) r_g^0. \end{aligned}$$

Complementarity:

$$(x+\alpha \Delta x)^T (s+\alpha \Delta s) + (\tau+\alpha \Delta \tau)(\kappa+\alpha \Delta \kappa) = (1-\alpha(1-\gamma))(x^T s+\tau \kappa).$$

Computing the scaling matrix

Theorem (Schnabel [3])

Let $S, Y \in \mathbb{R}^{n\times p}$ have full rank p. Then there exists $H \succ 0$ such that $HS = Y$ if and only if $Y^T S \succ 0$.

As a consequence

$$H = Y(Y^T S)^{-1} Y^T + ZZ^T$$

where $Z^T S = 0$, $\mathbf{rank}(Z) = n - p$. We have $n = 3$, $p = 2$ and

$$S := \begin{pmatrix} x & \tilde{x} \end{pmatrix}, \quad Y := \begin{pmatrix} s & \tilde{s} \end{pmatrix},$$

with

$$Y^T S = \begin{bmatrix} x^T s & \theta \\ \theta & \tilde{x}^T \tilde{s} \end{bmatrix}$$

and

$$\det(Y^T S) = \theta^2 \left((x^T s/\theta)(\tilde{x}^T \tilde{s}/\theta) - 1 \right) = \theta^2(\mu\tilde{\mu} - 1) \geq 0,$$

(equality only on the central path).

Computing the scaling matrix

The scaling for $n = 3$ satisfies

$$\begin{aligned} W^T W &= Y(Y^T S)^{-1} Y^T + zz^T \\ &= \frac{1}{\theta(\mu\tilde{\mu} - 1)} \left(\tilde{\mu} s s^T + \mu \tilde{s} \tilde{s}^T - s \tilde{s}^T - \tilde{s} s^T \right) + zz^T, \end{aligned}$$

with $\begin{pmatrix} x & \tilde{x} \end{pmatrix}^T z = 0$.

Expanding the BFGS update [3]

$$H^+ = H + Y(Y^T S)^{-1} Y^T - HS(S^T H S)^{-1} S^T H,$$

for $H \succ 0$ gives the scaling by Tunçel [4] and Myklebust [2], *i.e.*,

$$zz^T = H - HS(S^T H S)^{-1} S^T H.$$

Open issues for nonsymmetric cones

- As shown in [4, 2], we have polynomial-time complexity if

$$\xi^{-1} F''(x) \preceq W^T W \preceq \xi F_*''(s)^{-1}$$

  is bounded, *i.e.*, if $\xi = \mathcal{O}(1)$. We have no proof of this.
- Optimal scaling matrix. Currently we use a BFGS update.
- Higher-order Mehrotra-type corrector terms are illusive. We currently use

$$\Delta s + W^T W \Delta x = -s - \mu F'(x) - \frac{\mu}{2} F'''(x)[\Delta x_{\mathrm{aff}}, \Delta x_{\mathrm{aff}}],$$

  where Δx_{aff} is an affine-like search-step.

Logistic regression example

Given n binary training-points $\{(x_i, y_i)\}$.

Training:

$$
\begin{array}{ll}
\text{minimize} & \sum_i t_i + \lambda r \\
\text{subject to} & t_i \geq \log(1 + \exp(-\theta^T x_i)), \quad y_i = 1, \\
& t_i \geq \log(1 + \exp(\theta^T x_i)), \quad\;\; y_i = 0, \\
& r \geq \|\theta\|_2,
\end{array}
$$

$2n$ exponential cones $+$ 1 quadratic cone.

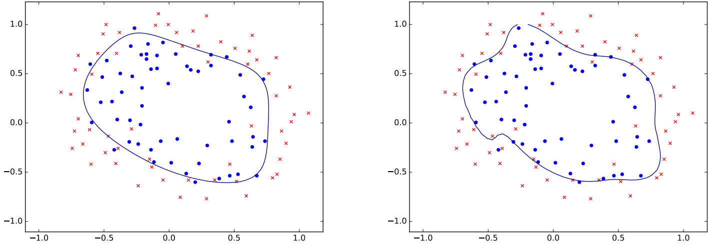
Classifier:

$$
h_\theta(z) = \frac{1}{1 + \exp(-\theta^T z)}.
$$

Logistic regression example

```python
from mosek.fusion import *

# t >= log( 1 + exp(u) )
def softplus(M, t, u):
    aux = M.variable(2)
    M.constraint(Expr.sum(aux), Domain.lessThan(1.0))
    M.constraint(Expr.hstack(aux, Expr.constTerm(2, 1.0),
                             Expr.vstack(Expr.sub(u,t), Expr.neg(t))),
                 Domain.inPExpCone())

# Model logistic regression (regularized with full 2-norm of theta)
# lambda - regularization parameter
def logisticRegression(X, y, lamb=1.0):
    n, d = X.shape          # num samples, dimension
    M = Model()
    theta = M.variable(d)
    t     = M.variable(n)
    reg   = M.variable()

    M.objective(ObjectiveSense.Minimize, Expr.add(Expr.sum(t), Expr.mul(lamb,reg)))
    M.constraint(Var.vstack(reg, theta), Domain.inQCone())

    for i in range(n):
        dot = Expr.dot(X[i], theta)
        if y[i]==1:
            softplus(M, t.index(i), Expr.neg(dot))
        else:
            softplus(M, t.index(i), dot)

    return M, theta
```

Logistic regression example



Decision regions for different regularizations. Data lifted to the space of degree 6 polynomials.

Logistic regression example

```
Optimizer  - threads                : 20
Optimizer  - solved problem         : the primal
Optimizer  - Constraints            : 236
Optimizer  - Cones                  : 237
Optimizer  - Scalar variables       : 855               conic                  : 737
Optimizer  - Semi-definite variables: 0                 scalarized             : 0
Factor     - setup time             : 0.00              dense det. time        : 0.00
Factor     - ML order time          : 0.00              GP order time          : 0.00
Factor     - nonzeros before factor : 7257              after factor           : 7257
Factor     - dense dim.             : 0                 flops                  : 9.66e+05
ITE PFEAS    DFEAS    GFEAS    PRSTATUS   POBJ              DOBJ              MU       TIME
0   1.6e+00  1.3e+00  9.9e+01  0.00e+00   9.768593109e+01   0.000000000e+00   1.0e+00  0.00
1   8.1e-01  6.6e-01  6.8e+01  6.60e-01   9.011469440e+01   3.552297591e+01   5.4e-01  0.01
2   3.2e-01  2.6e-01  4.3e+01  8.10e-01   7.052557003e+01   4.814005503e+01   2.2e-01  0.01
3   1.6e-01  1.3e-01  3.0e+01  9.51e-01   5.716944320e+01   4.630260918e+01   1.1e-01  0.01
4   7.2e-02  5.9e-02  2.0e+01  9.28e-01   4.754032019e+01   4.248014972e+01   5.2e-02  0.01
5   4.2e-02  3.4e-02  1.5e+01  8.70e-01   4.269747692e+01   3.971483592e+01   3.1e-02  0.01
6   2.5e-02  2.0e-02  1.1e+01  8.15e-01   3.929422749e+01   3.748825666e+01   1.9e-02  0.01
7   1.6e-02  1.3e-02  8.6e+00  7.54e-01   3.712558491e+01   3.593418437e+01   1.2e-02  0.01
8   9.3e-03  7.6e-03  6.4e+00  7.23e-01   3.535772247e+01   3.462356155e+01   7.5e-03  0.02
9   6.4e-03  5.2e-03  5.2e+00  7.14e-01   3.443934016e+01   3.391733535e+01   5.4e-03  0.02
10  5.0e-03  4.1e-03  4.6e+00  7.48e-01   3.396250049e+01   3.355009827e+01   4.3e-03  0.02
11  3.3e-03  2.7e-03  3.6e+00  7.22e-01   3.331083099e+01   3.303323369e+01   2.9e-03  0.02
12  2.7e-03  2.2e-03  3.2e+00  7.28e-01   3.302865568e+01   3.280278682e+01   2.4e-03  0.02
13  2.2e-03  1.8e-03  2.9e+00  7.56e-01   3.282977819e+01   3.264128094e+01   2.0e-03  0.02
14  1.5e-03  1.2e-03  2.3e+00  6.97e-01   3.247818711e+01   3.234470459e+01   1.5e-03  0.02
15  1.1e-03  8.9e-04  1.8e+00  6.52e-01   3.221441130e+01   3.211463097e+01   1.1e-03  0.02
16  9.3e-04  7.6e-04  1.6e+00  6.00e-01   3.210593508e+01   3.201793882e+01   9.4e-04  0.03
17  7.0e-04  5.7e-04  1.3e+00  5.24e-01   3.191120208e+01   3.184089496e+01   7.4e-04  0.03
18  5.3e-04  4.4e-04  1.1e+00  4.64e-01   3.174702006e+01   3.168994262e+01   5.8e-04  0.03
19  3.5e-04  2.9e-04  8.0e-01  4.37e-01   3.153180306e+01   3.149066395e+01   4.0e-04  0.03
20  2.4e-04  1.9e-04  6.1e-01  4.88e-01   3.136835364e+01   3.133901910e+01   2.8e-04  0.03
21  1.5e-04  1.3e-04  4.6e-01  5.95e-01   3.123979806e+01   3.122013885e+01   1.9e-04  0.03
22  8.1e-05  6.6e-05  3.1e-01  6.43e-01   3.110705011e+01   3.109619585e+01   1.0e-04  0.03
23  5.2e-05  4.3e-05  2.4e-01  7.72e-01   3.104953216e+01   3.104241448e+01   6.9e-05  0.04
24  3.3e-05  2.7e-05  1.8e-01  8.40e-01   3.100710801e+01   3.100267855e+01   4.4e-05  0.04
25  1.7e-05  1.4e-05  1.3e-01  8.85e-01   3.097269722e+01   3.097037756e+01   2.4e-05  0.04
26  4.1e-06  3.4e-06  5.8e-02  9.39e-01   3.094182106e+01   3.094128534e+01   6.0e-06  0.04
27  1.1e-06  9.1e-07  3.0e-02  9.84e-01   3.093399109e+01   3.093384551e+01   1.6e-06  0.04
28  8.5e-08  6.9e-08  8.1e-03  9.97e-01   3.093131789e+01   3.093130706e+01   1.3e-07  0.04
29  2.1e-08  1.7e-08  4.0e-03  1.00e+00   3.093115672e+01   3.093115404e+01   3.1e-08  0.04
30  5.7e-09  4.6e-09  2.1e-03  1.00e+00   3.093112004e+01   3.093111935e+01   8.8e-09  0.05
31  1.6e-09  5.4e-10  7.4e-04  1.00e+00   3.093110805e+01   3.093110797e+01   1.1e-09  0.05
```

Mixed-integer conic optimization

- Nonsymmetric conic solver extended with branch-and-bound.
- Preliminary work.
- Integer exp-cone test instances from CBLIB by Miles Lubin.

Mixed-integer exponential-cone instances I

Successfully solved instances

	Time	Obj. value	# nodes
syn40m04h	6.58	-901.75	476
syn40m03h	2.31	-395.15	276
syn40m02h	0.43	-388.77	14
syn40h	0.19	-67.713	16
syn30m04h	3.27	-865.72	450
syn30m03h	1.11	-654.16	165
syn30m02m	1091.4	-399.68	348085
syn30m02h	0.44	-399.68	58
syn30m	9.98	-138.16	7849
syn30h	0.13	-138.16	11
syn20m04m	1833.48	-3532.7	534769
syn20m04h	0.55	-3532.7	27
syn20m03m	300.47	-2647	118089
syn20m03h	0.37	-2647	25
syn20m02m	28.21	-1752.1	14321
syn20m02h	0.19	-1752.1	11
syn20m	0.63	-924.26	645
syn20h	0.09	-924.26	11
syn15m04m	16.59	-4937.5	5567
syn15m04h	0.33	-4937.5	7
syn15m03m	4.77	-3850.2	1907
syn15m03h	0.19	-3850.2	5
syn15m02m	1.24	-2832.7	751
syn15m02h	0.11	-2832.7	5
syn15m	0.12	-853.28	85
syn15h	0.04	-853.28	3
syn10m04m	2.99	-4557.1	1983
syn10m04h	0.16	-4557.1	5

Mixed-integer exponential-cone instances II

Successfully solved instances

syn10m03m	1.13	-3354.7	923
syn10m03h	0.11	-3354.7	5
syn10m02m	0.36	-2310.3	409
syn10m02h	0.08	-2310.3	5
syn10m	0.05	-1267.4	31
syn10h	0	-1267.4	0
syn05m04m	0.17	-5510.4	45
syn05m04h	0.06	-5510.4	3
syn05m03m	0.09	-4027.4	33
syn05m03h	0.04	-4027.4	3
syn05m02m	0.06	-3032.7	23
syn05m02h	0.03	-3032.7	3
syn05m	0.02	-837.73	11
syn05h	0.02	-837.73	5
rsyn0840m04h	39.28	-2564.5	2197
rsyn0840m03h	15.34	-2742.6	1577
rsyn0840m02h	1.56	-734.98	149
rsyn0840h	0.27	-325.55	19
rsyn0830m04h	29.9	-2529.1	2115
rsyn0830m03h	8.3	-1543.1	935
rsyn0830m02h	2.38	-730.51	299
rsyn0830m	227.14	-510.07	99495
rsyn0830h	0.44	-510.07	117
rsyn0820m04h	10.59	-2450.8	635
rsyn0820m03h	18.16	-2028.8	2079
rsyn0820m02h	3.35	-1092.1	510
rsyn0820m	110.08	-1150.3	58607
rsyn0820h	0.46	-1150.3	145
rsyn0815m04h	5.79	-3410.9	587
rsyn0815m03h	7.37	-2827.9	866

Mixed-integer exponential-cone instances III

Successfully solved instances

rsyn0815m02m	2345.68	-1774.4	567030
rsyn0815m02h	2.08	-1774.4	365
rsyn0815m	10.47	-1269.9	7059
rsyn0815h	0.36	-1269.9	238
rsyn0810m04h	6.95	-6581.9	677
rsyn0810m03h	4.95	-2722.4	740
rsyn0810m02m	1353.22	-1741.4	425403
rsyn0810m02h	1.15	-1741.4	159
rsyn0810m	8.31	-1721.4	9041
rsyn0810h	0.21	-1721.4	134
rsyn0805m04m	578.5	-7174.2	66975
rsyn0805m04h	1.92	-7174.2	101
rsyn0805m03m	186.01	-3068.9	37908
rsyn0805m03h	1.61	-3068.9	177
rsyn0805m02m	86.81	-2238.4	34126
rsyn0805m02h	0.87	-2238.4	201
rsyn0805m	3.16	-1296.1	4639
rsyn0805h	0.19	-1296.1	120

Mixed-integer exponential-cone instances

Timed-out instances

	Time	Obj. value	# nodes
gams01	3600.0	22265	70232
rsyn0810m03m	3600.0	-2722.4	493926
rsyn0810m04m	3600.0	-6580.9	307231
rsyn0815m03m	3600.1	-2827.9	420782
rsyn0815m04m	3600.2	-3359.8	309729
rsyn0820m02m	3600.2	-1077.6	683356
rsyn0820m03m	3600.2	-1980.4	380611
rsyn0820m04m	3600.1	-2401.1	262880
rsyn0830m02m	3600.4	-705.46	568113
rsyn0830m03m	3600.2	-1456.3	368794
rsyn0830m04m	3600.1	-2395.7	206456
rsyn0840m	3600.3	-325.55	1157426
rsyn0840m02m	3600.5	-634.17	422224
rsyn0840m03m	3600.1	-2656.5	252651
rsyn0840m04m	3600.0	-2426.3	142895
syn30m03m	3600.2	-654.15	831798
syn30m04m	3600.2	-848.07	643266
syn40m02m	3600.2	-366.77	748603
syn40m03m	3600.3	-355.64	607359
syn40m04m	3600.2	-859.71	371521

Further information

- Documentation: `https://www.mosek.com/documentation/`
  - Manuals for interfaces.
  - Modeling cook-book.
  - White papers.
- Examples and tutorials:
  - `https://github.com/MOSEK/Tutorials`

References

[1] M. Lubin and E. Yamangil and R. Bent and J. P. Vielma.
Extended Formulations in Mixed-integer Convex Programming.
Technical report, 2016.
arXiv:1511.06710v3.

[2] T. Myklebust and L. Tunçel.
Interior-point algorithms for convex optimization based on primal-dual metrics.
Technical report, 2014.

[3] R. B. Schnabel.
Quasi-newton methods using multiple secant equations.
Technical report, Colorado Univ., Boulder, Dept. Comp. Sci., 1983.

[4] L. Tunçel.
Generalization of primal-dual interior-point methods to convex optimization problems in conic form.
Foundations of Computational Mathematics, 1:229–254, 2001.