Semidefinite Programming for Power System Stability and Optimization

MOSEK Workshop on Semidefinite Optimization in Power Flow Problems

Spyros Chatzivasileiadis

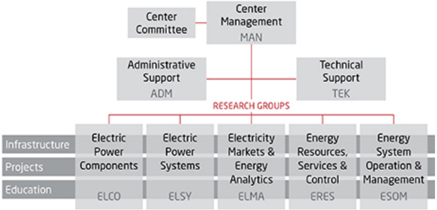
DTU Center for Electric Power and Energy

Outline

- Who are we? ... And what do we do?
- What is semidefinite programming (SDP) ?
- Power System Stability Assessment and SDP
- Power System Optimization and SDP

DTU Center for Electric Power and Energy

- **Established 15 Aug. 2012; merger of existing units (Lyngby+Risø)**
- One of the strongest university centers in Europe with $\sim$ 100 employees



Mission: Provides cutting-edge research, education and innovation in the field of electric power and energy to meet the future needs of society regarding a reliable, cost efficient and environmentally friendly energy system

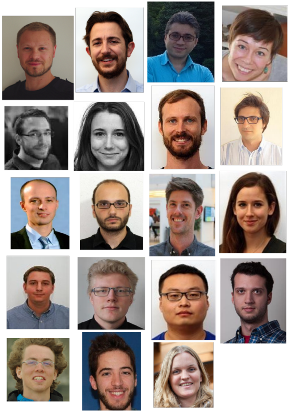
- **BSc & MSc:** Electrical Engineering, Wind Energy, Sustainable Energy
- **Direct Support from:** Energinet.dk, Siemens, DONG Energy, Danfoss

DTU ranked world 2nd in Energy Science and Engineering[1]

[1] Shanghai Ranking 2016, Global Ranking of Academic Subjects

The Energy Analytics & Markets group

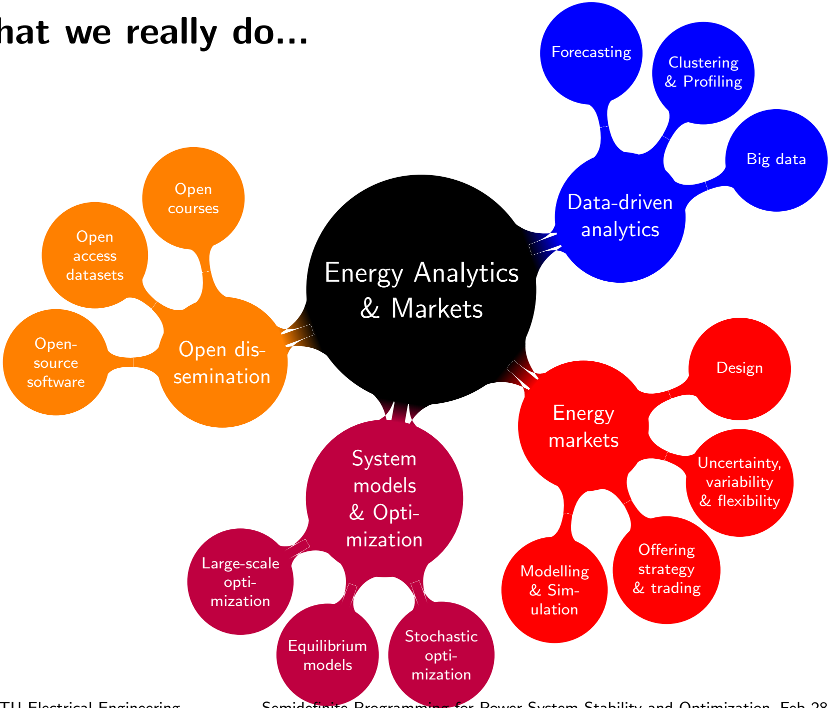
One of the 5 groups of the Center for Electric Power and Energy,
Department of Electrical Engineering



- **Resources:** (10 nationalities)
  - *Faculty:* 1 Prof, 2 Assist. Profs.
  - *Junior:* 3 post-doc fellows, 9 Ph.D. students (+2 externals), 2-3 research assistants
  - + student helpers, and Ph.D. guests from China, Brazil, US, Spain, France, Italy, Netherlands, Germany, etc.
- **Projects** (active in 2016):
  - **EU:** BestPaths
  - **Danish:** 5s, EcoGrid 2.0, CITIES, EnergyLab Nordhavn, EnergyBlock, CORE, MULTI-DC
  - **Danish-Chinese:** PROAIN

- **Education:** Various courses on renewables forecasting, optimization, and electricity markets
- (hopefully) recognized leading expertise in energy analytics and markets

What we really do...



Research Topics (Selection)

- Optimal operation of combined heat, gas, and electricity networks
- Game theoretical approaches for electricity market participants
- Spatiotemporal forecasting for wind, solar, and energy demand
- Stochastic electricity market design and value of information
- HVDC optimization and control under uncertainty

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What is Semidefinite Programming? (SDP)

- SDP is the "generalized" form of an LP (linear program)

Linear Programming

$$\min c^T \cdot x$$

subject to:

$$a_i \cdot x = b_i, \quad i = 1, \ldots, m$$
$$x \geq 0, \quad x \in R^n$$

Semidefinite Programming

$$\min C \bullet X := \sum_i \sum_j C_{ij} X_{ij}$$

subject to:

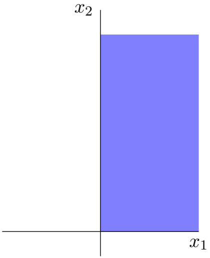
$$A_i \bullet X = b_i, \quad i = 1, \ldots, m$$
$$X \succeq 0$$

- LP: Optimization variables in the form of a vector x.
- SDP: Optim. variables in the form of a positive semidefinite *matrix* X.
- SDP=LP: for diagonal matrices

Example: Feasible space of SDP vs LP variables

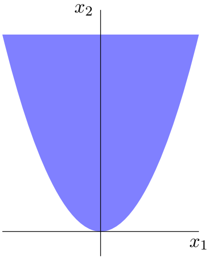
LP

$$x_1 \geq 0$$
$$x_2 \geq 0$$



SDP

$$X = \begin{bmatrix} x_2 & x_1 \\ x_1 & 1 \end{bmatrix} \succeq 0 \Rightarrow x_2 - x_1^2 \geq 0$$



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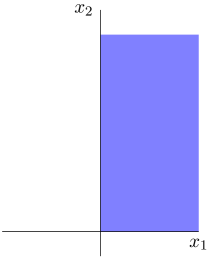

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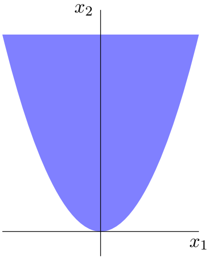


- In SDP we can express quadratic constraints, e.g. x_1^2 or $x_1 x_2$
- optimization variables need not be strictly non-negative
- LP is a special case of SDP

SDP for Power System Stability and Optimization

SDP for Power System Stability

find a feasible X

subject to:

$$A_i \bullet X \succeq 0$$
$$X \succeq 0$$

SDP for Optimal Power Flow

minimize cost of electricity

$$\min C \bullet X$$

subject to:

voltage and power flow constraints

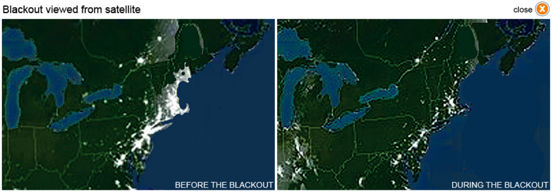
$$A_i \bullet X = b_i, \quad i = 1, \ldots, m$$
$$X \succeq 0$$

Robust Power System Stability Assessment with Extensions to Inertia and Topology Control

work with:
Thanh Long Vu, Kostya Turitsyn
MIT Mechanical Engineering

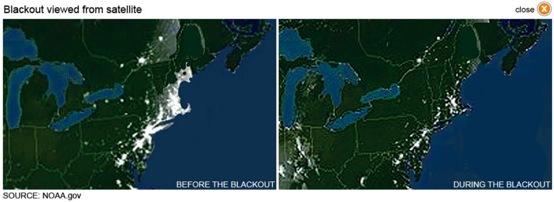
Power blackouts

Blackout viewed from satellite



SOURCE: NOAA.gov

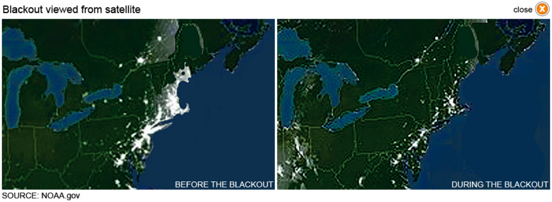
Power blackouts



Statistics:

- 2003 US: 55M people; 2011 India: 700M people
- Frequency: $\approx 1hr/year \implies$ economic damage: $\approx 100B\$/year$
- Total electric energy cost in US: $\approx 400B\$/year$

Power blackouts



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Challenges and opportunities:

- New algorithms for better decision-making

Dynamic Security Assessment

- Security = ability to withstand disturbances
- Security Assessment:
  - Screen contingency list every 15 mins
  - Prepare contingency plans for critical scenarios.

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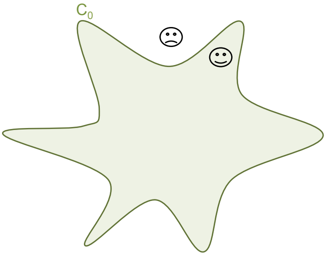
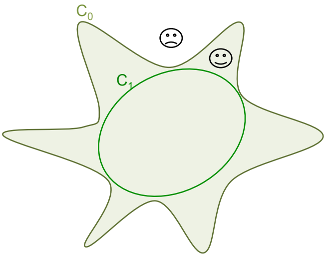
- Dynamic simulations are hard:
  - DAE system with about 10k degrees of freedom
  - Faster than real-time simulations is an open research topic
- Alternative: Energy methods = Security certificates

Security certificates



- Security region: non-convex, NP-hard characterization

Security certificates



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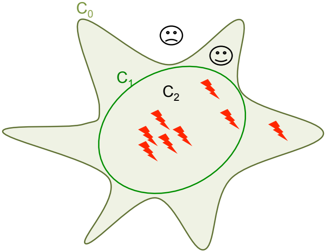
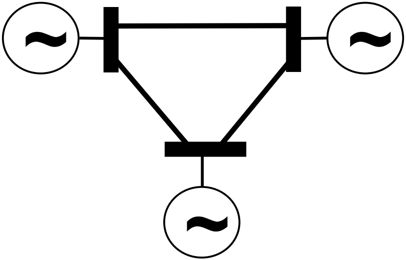
- Security certificates: tractable sufficient conditions

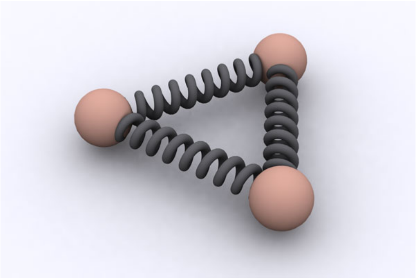
Security certificates



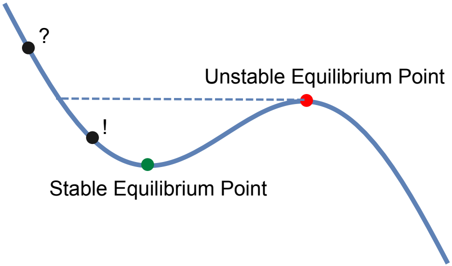
- Security region: non-convex, NP-hard characterization
- Security certificates: tractable sufficient conditions
- Strategy: certify security of most of scenarios with conservative conditions, use simulations for few really dangerous scenarios

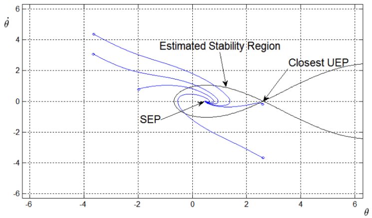
Closest mechanical equivalent to a power system is a mass-spring system





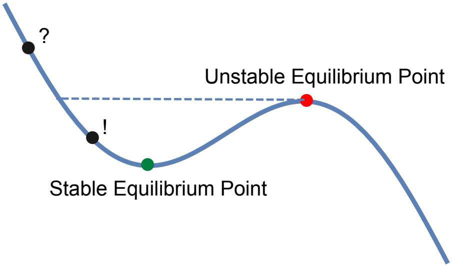
Energy method

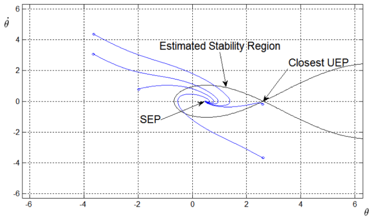




- If $E(\delta_0, \dot{\delta}_0) < E_{CUEP}$, then stable

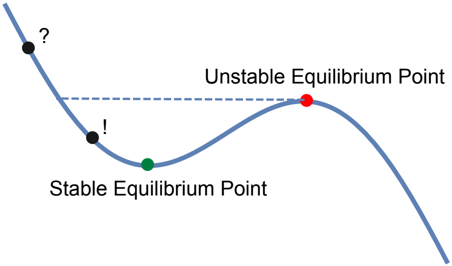
Energy method

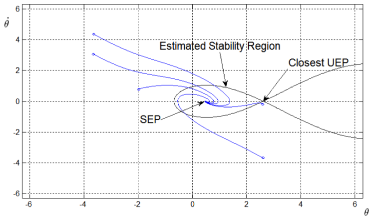




- If $E(\delta_0, \dot{\delta}_0) < E_{CUEP}$, then stable
- Fast transient stability certificate

Energy method





- If $E(\delta_0, \dot{\delta}_0) < E_{CUEP}$, then stable
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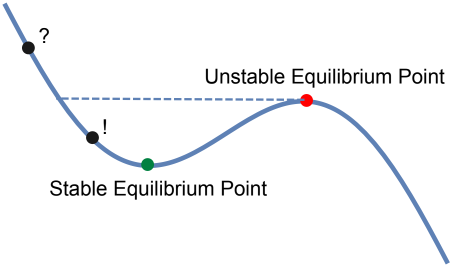
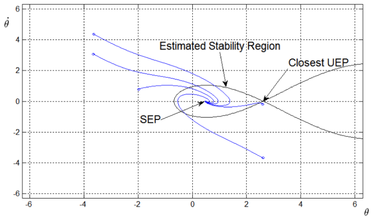
- Computing E_{CUEP} is an NP-hard problem

Energy method





- If $E(\delta_0, \dot{\delta}_0) < E_{CUEP}$, then stable
- Fast transient stability certificate
- Computing E_{CUEP} is an NP-hard problem
- Certificates are generally conservative

Modeling Approach

- Non-linear swing equation

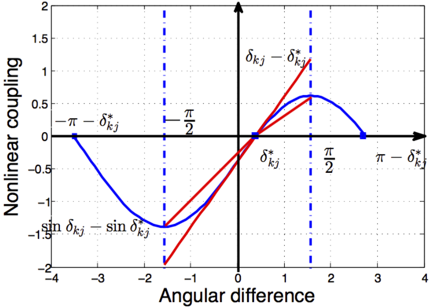
$$m_k\ddot{\delta}_k + d_k\dot{\delta}_k + \sum_{j\in\mathcal{N}_k} a_{kj}\sin(\delta_k - \delta_j) = P_k \tag{1}$$

$$m_k\ddot{\delta}_k + d_k\dot{\delta}_k + \sum_{j\in\mathcal{N}_k} a_{kj}(\sin(\delta_{kj}) - \sin(\delta^*_{kj})) = 0 \tag{2}$$

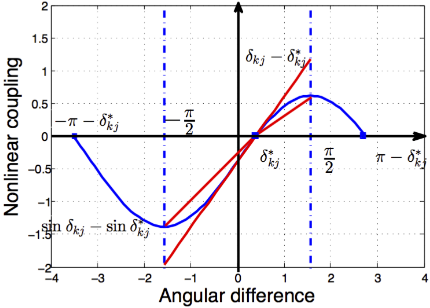
$$\dot{x} = Ax - BF(Cx) \tag{3}$$

- Structure-preserving model: A and B do not correspond to the reduced model
- $x = \delta_i - \delta_i^*$
- A, B, C are independent of the operating point P_k
- $F(Cx)$ stands for the non-linear function $sin(\delta_{kj}) - sin(\delta^*_{kj})$

Bounding nonlinearity



Bounding nonlinearity



- Sector bound on nonlinearity for polytope $\mathcal{P} : \{\delta, \dot{\delta} : |\delta_{kj}| < \frac{\pi}{2}\}$

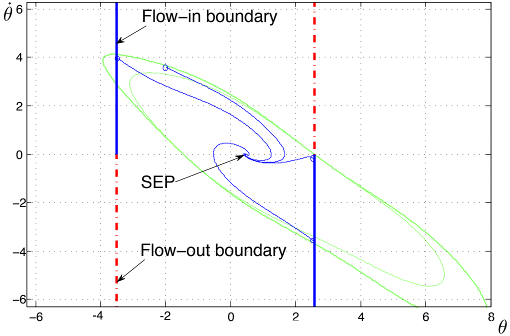
Stability certificate

- If:

$$\bar{A}^T P + P\bar{A} + \frac{(1-g)^2}{4} C^T C + PBB^T P \preceq 0 \tag{4}$$

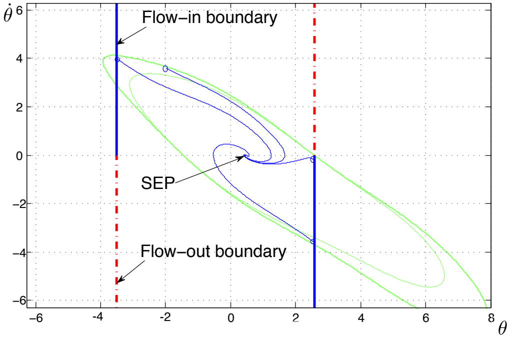
- there exists a quadratic Lyapunov function $V = x^T P x$ that is decreasing whenever $x(t) \in \mathcal{P}$.

Stability region



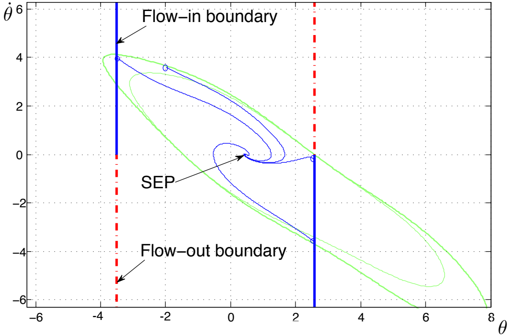
- Lyapunov function $x^T P x$ is an ellipsoid

Stability region



- Lyapunov function $x^T Px$ is an ellipsoid
- Due to the sector bound on the nonlinear $\sin()$ term, stability is certified only as long as we stay within $[-\pi/2, \pi/2]$

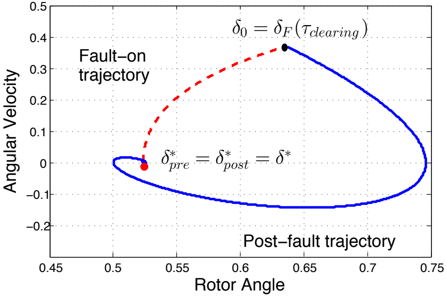
Stability region



- Lyapunov function $x^T P x$ is an ellipsoid
- Due to the sector bound on the nonlinear $\sin()$ term, stability is certified only as long as we stay within $[-\pi/2, \pi/2]$
- Finding the V_{min} within these bounds is now a convex problem!
  - We can solve (even large) convex problems fast and efficiently

Extensions to Remedial Actions

- Can incorporate inertia and damping control by appropriately changing A and B $\Rightarrow$ bound the growth of Lyapunov function



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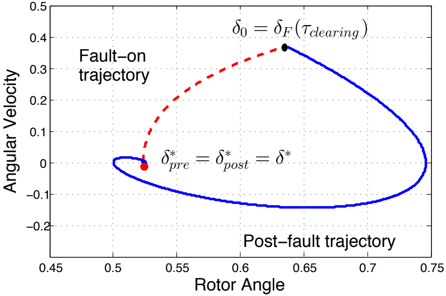
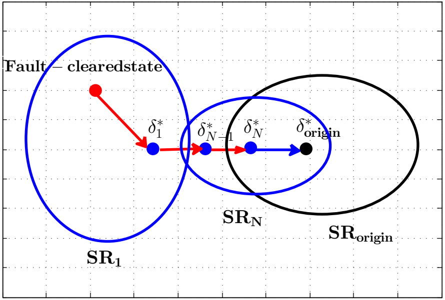


- Can incorporate topology control, e.g. FACTS, by appropriately changing A and B $\Rightarrow$ generate a set of ellipsoids that will guarantee the convergence of x_0 to the post-fault equilibrium



Convex Relaxations of Chance Constrained AC Optimal Power Flow

work with:
Andreas Venzke

The Optimal Power Flow Problem (OPF)

minimize the cost of electricity generation

subject to:

demand of electric loads

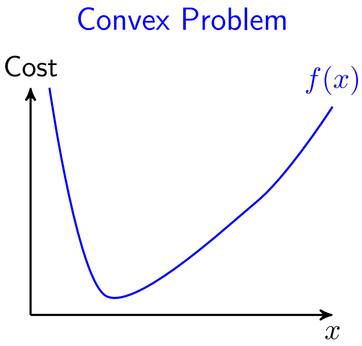
maximum power of generators

maximum power capacity of transmission lines

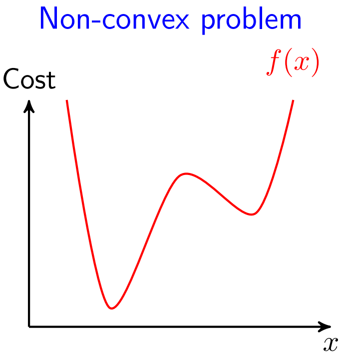
voltage limits

- The problem is:
  - non-linear: power flow depends on the square of voltages
  - non-convex: there are more than one (local) minima

Convex vs. Non-convex Problem



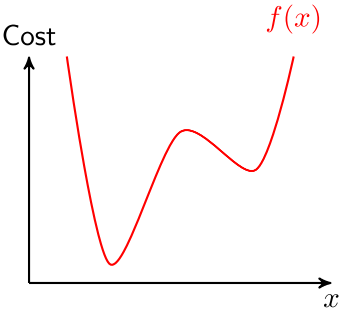
One global minimum



Several local minima

Several local minima: So what?

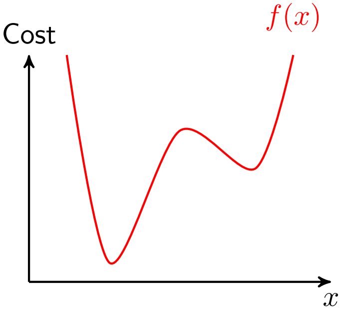
- Electricity Markets: Assume that the difference in the cost function of a local minimum versus a global minimum is 2%
- The total electric energy cost in the US is $\approx$ 400 Billion\$/year
- 2% amounts to 8 billion US\$ in economic losses per year



- Technical operation: Convex OPF determines absolute lower or upper bound of control effort $\rightarrow$ useful in branch-and-bound methods for mixed integer programming, e.g. unit commitment, capacitor switching
- Convex problems guarantee that we find a global minimum $\rightarrow$ convexify the OPF problem

Convexifying the Optimal Power Flow problem (OPF)

- Convex relaxation transforms OPF to convex Semi-Definite Program (SDP)



Convex Relaxation

[2] Javad Lavaei and Steven H Low. "Zero duality gap in optimal power flow problem". In: *IEEE Transactions on Power Systems* 27.1 (2012), pp. 92–107

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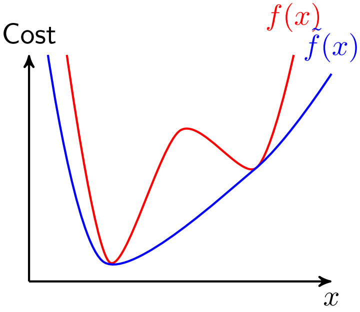

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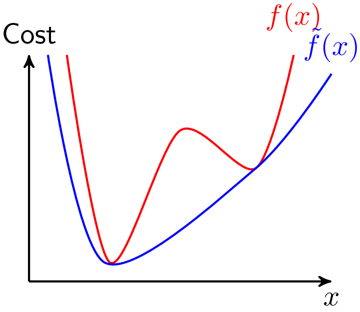
Convexifying the Optimal Power Flow problem (OPF)

- Convex relaxation transforms OPF to convex Semi-Definite Program (SDP)
- Under certain conditions, the obtained solution is the global optimum to the original OPF problem[2]



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Transforming the AC-OPF to an SDP

- Power is a quadratic function of voltage, e.g.: $P_{ij} = f(V_i^2, V_j^2, V_i V_j)$

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Transforming the AC-OPF to an SDP

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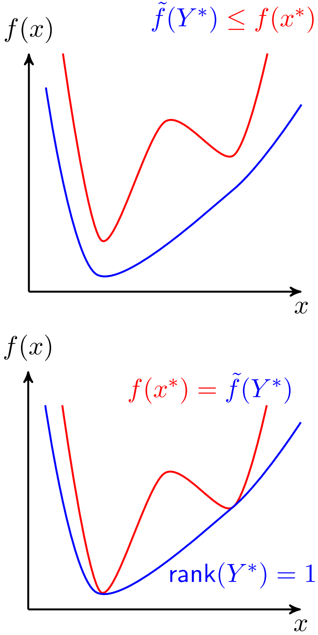
- Let $W = VV^T$ and express $P = f(W)$. In that case, P is an affine function of W.
- If $W \succeq 0$ and $\text{rank}(W) = 1$:

  W can be expressed as a product of vectors and we can recover the solution V to our original problem
- However the rank-1 constraint is non-convex...

Applying convex relaxations with SDP



EXACT: $W = VV^T$

$\Downarrow$

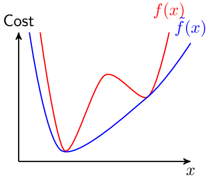
RELAX: $W \succeq 0$

~~$\text{rank}(W) = 1$~~

- For the objective functions, it holds EXACT $\geq$ RELAX
- The RELAX problem is an SDP problem!
- If W^* happens also to be rank-1, then EXACT $=$ RELAX!

Notes on the Convex Relaxation

- Relaxation gap: Difference between the solution of original non-convex, non-linear OPF and the SDP



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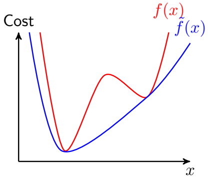

- If $\text{rank}(W) = 1$ or 2: solution to original OPF problem can be recovered $\rightarrow$ global optimum
- If $\text{rank}(W) \geq 3$: the solution W has no physical meaning (but still it is a lower bound)
- Molzahn[3] derives a heuristic rule: if the ratio of the 2nd to the 3rd eigenvalue of W is larger than 10^5 $\rightarrow$ we obtain rank-2.

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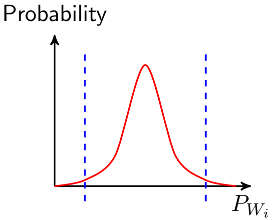
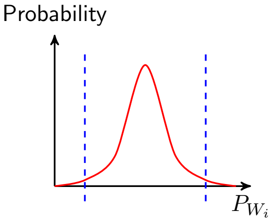
Introducing Uncertainty

- Increasing share of uncertain renewables
  $\Rightarrow$ Include chance constraints in OPF: Constraints should be fullfilled for a defined probability ϵ, given an underlying distribution of the uncertainty



Introducing Uncertainty

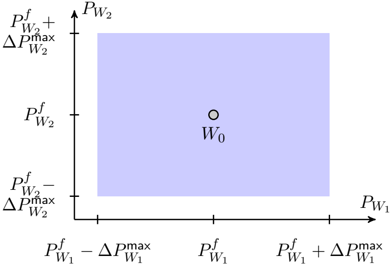


- Increasing share of uncertain renewables
  $\Rightarrow$ Include chance constraints in OPF:
  Constraints should be fullfilled for a defined probability ϵ, given an underlying distribution of the uncertainty

- Uncertainty in wind forecast errors
- Our Goal: Convex Chance-Constrained AC-OPF
- Pros:
  - Can consider losses and large uncertainty deviations
  - Considers reactive power $\rightarrow$ reactive power flow control
  - Convex $\rightarrow$ can find global optimum
- Cons:
  - Scalable?

Uncertainty Sets - Rectangular & Gaussian

How to model the uncertainty distribution of forecast errors ΔP_{W_i}?



Rectangular uncertainty set: General non-Gaussian distributions. Upper and lower bounds are known a-priori.

[3] M. Vrakopoulou, M. Katsampani, K. Margellos, J. Lygeros, G. Andersson. "Probabilistic security-constrained AC optimal power flow". In: *IEEE PowerTech (POWERTECH)*. Grenoble, France, 2012.

Uncertainty Sets - Rectangular & Gaussian

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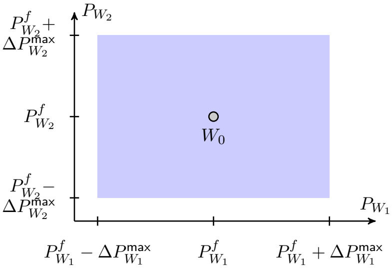
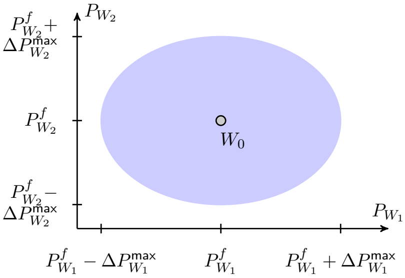


Rectangular uncertainty set: General non-Gaussian distributions. Upper and lower bounds are known a-priori.

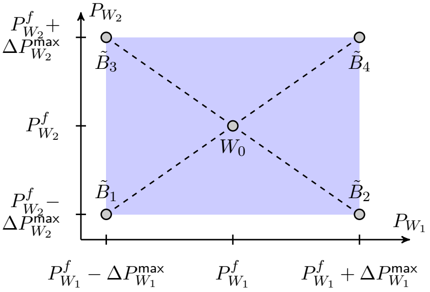


Ellipsoid uncertainty set: Multivariate Gaussian distribution with known standard deviation and confidence interval ϵ.

- First steps taken in Vrakopoulou et al, 2013. Here we extend this work in several ways.

[3] M. Vrakopoulou, M. Katsampani, K. Margellos, J. Lygeros, G. Andersson. "Probabilistic security-constrained AC optimal power flow". In: *IEEE PowerTech (POWERTECH)*. Grenoble, France, 2012.

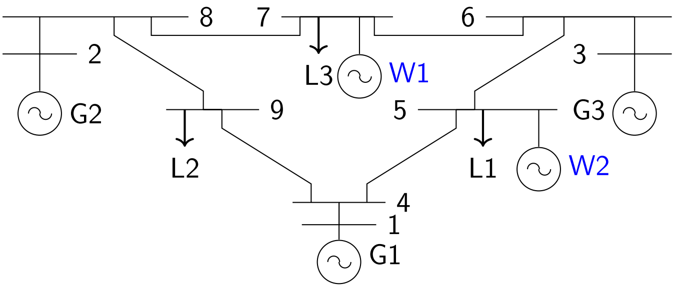
Formulation for Rectangular Uncertainty Set



- It suffices to enforce the chance constraints at the vertices v of the uncertainty set[4].

[4] Kostas Margellos, Paul Goulart, and John Lygeros. "On the road between robust optimization and the scenario approach for chance constrained optimization problems". In: *IEEE Transactions on Automatic Control* 59.8 (2014), pp. 2258–2263.

Test System



Modified IEEE 9-bus system with wind farms W1 and W2

- W1 with ± 50 MW deviation inside confidence interval
- W2 with ± 40 MW deviation inside confidence interval
- SDP-Solver: MOSEK v8
- Coded with Julia (open-source)

Simulation Results

Affine Policy for Rectangular Uncertainty Set

Generator droops	d_1 = [0.5 0.25 0.25 0 -1 0 0 0 0]
Generator droops	d_2 = [0.5 0.25 0.25 0 0 0 -1 0 0]
Weight power loss	$\mu = 0.4\ \frac{\$}{h\text{MW}}$
Generator cost	3378.73 $\frac{\$}{h}$
Eigenvalue ratios	$\rho(W_0) = 6.4 \times 10^6$
	$\rho^*(W_0 + \Delta\tilde{P}_1^{\max}\tilde{B}_1) = 2.5 \times 10^5$
	$\rho^*(W_0 + \Delta\tilde{P}_2^{\max}\tilde{B}_2) = 2.4 \times 10^5$
	$\rho^*(W_0 + \Delta\tilde{P}_3^{\max}\tilde{B}_3) = 2.7 \times 10^6$
	$\rho^*(W_0 + \Delta\tilde{P}_4^{\max}\tilde{B}_4) = 1.9 \times 10^6$

# Gen	V_G [p.u.]	P_G [MW]	Q_G [Mvar]	V_G^* [p.u.]	P_G^* [MW]	Q_G^* [Mvar]
G1	1.10	64.70	8.09	1.07	60.96	31.00
G2	1.09	97.21	-12.17	1.10	95.34	32.70
G3	1.08	65.43	-32.98	0.97	63.56	-80.45
W1	—	50.00	11.45	—	100.00	22.94
W2	—	40.00	1.39	—	0.00	0.00
$\sum$	—	317.34	-24.23	—	319.86	6.18
# Branch	from	to	P_{lm} [MW]	P_{lm}^* [MW]	Q_{lm} [Mvar]	Q_{lm}^* [Mvar]
3	5	6	42.87	67.50	-24.07	-35.04
Maximum voltage [p.u.]			$V^{\max}$	1.100	$(V^{\max})^*$	1.100

- we satisfy the conditions to obtain the global optimum
- all constraints are satisfied
- we find the true global minimum

Ongoing Work

- Convex formulation for chance-constrained AC-OPF[5]
- Investigating the conditions to obtain zero relaxation gap
- Investigating how to achieve scalability
- Extending this formulation to combined AC and HVDC grids

[5] A. Venzke, L. Halilbasic, U. Markovic, G. Hug, and S. Chatzivasileiadis. *Convex Relaxations of Chance Constrained AC Optimal Power Flow*. Submitted. [Online]: arxiv.org/abs/1702.08372. 2017

Conclusions

- "Semidefinite programming is the most exciting development in mathematical programming in the 1990's"[6]

- Power interruptions are extremely costly; secure operation is challenging
  - SDP-based methods can extract less conservative stability certificates

- Large systems have high costs ⇒ cannot afford to find a suboptimal local minimum
  - SDP-based optimization allows to recover the global optimum
  - We introduced convex relaxations for a chance-constrained AC-OPF

- Challenges: Numerics & scalability

[6] Robert M. Freund. *Introduction to Semidefinite Programming*. MIT Lecture Notes. 2009.

Thank you!

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References:

T.L. Vu and K. Turitsyn. "A Framework for Robust Assessment of Power Grid Stability and Resiliency". In: *IEEE Transactions on Automatic Control* PP.99 (2016), pp. 1–1. ISSN: 0018-9286. DOI: 10.1109/TAC.2016.2579743

S. Chatzivasileiadis, T.L. Vu, and K. Turitsyn. "Remedial Actions to Enhance Stability of Low-Inertia Systems". In: *IEEE Power and Energy Society General Meeting 2016, Boston, MA, USA*. 2016, pp. 1 –5

T.L. Vu, S. Chatzivasileiadis, H.D. Chang, and K. Turitsyn. "Structural Emergency Control Paradigm". In: *IEEE Journal on Emerging and Selected Topics in Circuits and Systems* (2017). Accepted. [Online]: arxiv.org/abs/1607.08183

A. Venzke, L. Halilbasic, U. Markovic, G. Hug, and S. Chatzivasileiadis. *Convex Relaxations of Chance Constrained AC Optimal Power Flow*. Submitted. [Online]: arxiv.org/abs/1702.08372. 2017