

## Cones

Quadratic cone  $\mathcal{Q}^n$

$$x_1 \geq \sqrt{x_2^2 + \dots + x_n^2}$$

Rotated quadratic cone  $\mathcal{Q}_r^n$

$$2x_1x_2 \geq x_3^2 + \dots + x_n^2, x_1, x_2 \geq 0$$

Power cone  $\mathcal{P}_3^{\alpha, 1-\alpha}$ ,  $\alpha \in (0, 1)$

$$x_1^\alpha x_2^{1-\alpha} \geq |x_3|, x_1, x_2 \geq 0$$

Exponential cone  $K_{\text{exp}}$

$$x_1 \geq x_2 e^{x_3/x_2}, x_2 \geq 0$$

## Simple bounds

$t \geq x^2$	$(0.5, t, x) \in \mathcal{Q}_r^3$
$ t  \leq \sqrt{x}$	$(0.5, x, t) \in \mathcal{Q}_r^3$
$t \geq  x $	$(t, x) \in \mathcal{Q}^2$
$t \geq 1/x, x > 0$	$(x, t, \sqrt{2}) \in \mathcal{Q}_r^3$
$t \geq  x ^p, p > 1$	$(t, 1, x) \in \mathcal{P}_3^{1/p, 1-1/p}$
$t \geq 1/x^p, x > 0, p > 0$	$(t, x, 1) \in \mathcal{P}_3^{1/(1+p), p/(1+p)}$
$ t  \leq x^p, x > 0, p \in (0, 1)$	$(x, 1, t) \in \mathcal{P}_3^{p, 1-p}$
$t \geq  x ^p/y^{p-1}, y \geq 0, p > 1$	$(t, y, x) \in \mathcal{P}_3^{1/p, 1-1/p}$
$t \geq x^T x/y, y \geq 0$	$(0.5t, y, x) \in \mathcal{Q}_r^{n+2}$
$t \geq e^x$	$(t, 1, x) \in K_{\text{exp}}$
$t \leq \log x$	$(x, 1, t) \in K_{\text{exp}}$
$t \geq 1/\log x, x > 1$	$(u, t, \sqrt{2}) \in \mathcal{Q}_r^3$ $(x, 1, u) \in K_{\text{exp}}$
$t \geq a_1^{x_1} \dots a_n^{x_n}, a_i > 0$	$(t, 1, \sum x_i \log a_i) \in K_{\text{exp}}$
$t \geq x e^x, x \geq 0$	$(t, x, u) \in K_{\text{exp}}$ $(0.5, u, x) \in \mathcal{Q}_r^3$
$t \geq \log(1 + e^x)$	$u + v \leq 1$ $(u, 1, x - t) \in K_{\text{exp}}$ $(v, 1, -t) \in K_{\text{exp}}$
$t \geq  x ^{3/2}$	$(t, 1, x) \in \mathcal{P}_3^{2/3, 1/3}$
$t \geq x^{3/2}, x \geq 0$	$(s, t, x), (x, 1/8, s) \in \mathcal{Q}_r^3$
$t \geq 1/x^3, x > 0$	$(t, x, 1) \in \mathcal{P}_3^{3/4, 1/4}$
$0 \leq t \leq x^{2/5}, x \geq 0$	$(x, 1, t) \in \mathcal{P}_3^{2/5, 3/5}, t \geq 0$

## Means and averaging

Log-sum-exp $t \geq \log(\sum e^{x_i})$	$(z_i, 1, x_i - t) \in K_{\text{exp}}$ $i = 1, \dots, n$
Harmonic mean $0 \leq t \leq n(\sum x_i^{-1})^{-1}$ $x_i > 0$	$\sum z_i \leq 1$ $(z_i, x_i, t) \in \mathcal{Q}_r^3$ $i = 1, \dots, n$ $\sum z_i = nt/2$
Geometric mean $ t  \leq (x_1 \dots x_n)^{1/n}$ $x_i > 0$	$(z_i, x_i, z_{i+1}) \in \mathcal{P}_3^{1-1/i, 1/i}$ $i = 2, \dots, n$ $z_2 = x_1, z_{n+1} = t$
Weighted geom. mean $ t  \leq x_1^{\alpha_1} \dots x_n^{\alpha_n}, x_i > 0$ $\alpha_i > 0, \sum \alpha_i = 1$	$(z_i, x_i, z_{i+1}) \in \mathcal{P}_3^{1-\beta_i, \beta_i}$ $\beta_i = \alpha_i/(\alpha_1 + \dots + \alpha_i)$ $i = 2, \dots, n$ $z_2 = x_1, z_{n+1} = t$
$ t  \leq x^{1/4} y^{5/12} z^{1/3}$ $x, y, z \geq 0$	$(s, z, t) \in \mathcal{P}_3^{2/3, 1/3}$ $(x, y, s) \in \mathcal{P}_3^{3/8, 5/8}$

## Entropy

$t \leq -x \log x$	$(1, x, t) \in K_{\text{exp}}$
$t \geq x \log(x/y)$	$(y, x, -t) \in K_{\text{exp}}$
$t \geq \log(1 + 1/x)$ $x > 0$	$(x + 1, u, \sqrt{2}) \in \mathcal{Q}_r^3$ $(1 - u, 1, -t) \in K_{\text{exp}}$
$t \leq \log(1 - 1/x)$ $x > 1$	$(x, u, \sqrt{2}) \in \mathcal{Q}_r^3$ $(1 - u, 1, t) \in K_{\text{exp}}$
$t \geq x \log(1 + x/y)$ $x, y > 0$	$(y, x + y, u) \in K_{\text{exp}}$ $(x + y, y, v) \in K_{\text{exp}}$ $t + u + v = 0$

## Convex quadratic problems

Let $\Sigma \in \mathbb{R}^{n \times n}$ , symmetric, p.s.d. Find $\Sigma = LL^T$ , $L \in \mathbb{R}^{n \times k}$ (Cholesky factor). Then $x^T \Sigma x = \ L^T x\ _2^2$ .	
$t \geq \frac{1}{2} x^T \Sigma x$	$(1, t, L^T x) \in \mathcal{Q}_r^{k+2}$
$t \geq \sqrt{x^T \Sigma x}$	$(t, L^T x) \in \mathcal{Q}^{k+1}$
$\frac{1}{2} x^T \Sigma x + p^T x + q \leq 0$	$(1, -p^T x - q, L^T x) \in \mathcal{Q}_r^{k+2}$
$\max_x c^T x - \frac{1}{2} x^T \Sigma x$	$\max c^T x - r$ $(1, r, L^T x) \in \mathcal{Q}_r^{k+2}$
$c^T x + d \geq \ Ax + b\ _2$	$(c^T x + d, Ax + b) \in \mathcal{Q}^{m+1}$

## Norms, $x \in \mathbb{R}^n$

$\ \cdot\ _1, t \geq \sum  x_i $	$(z_i, x_i) \in \mathcal{Q}^2, t = \sum z_i$
$\ \cdot\ _2, t \geq (\sum x_i^2)^{1/2}$	$(t, x) \in \mathcal{Q}^{n+1}$
$\ \cdot\ _p, p > 1$ $t \geq (\sum  x_i ^p)^{1/p}$	$(z_i, t, x_i) \in \mathcal{P}_3^{1/p, 1-1/p}$ $i = 1, \dots, n$ $\sum z_i = t$

## Geometry

Bounding ball	$\min r$
$\min_x \max_i \ x - x_i\ _2$	$(r, x - x_i) \in \mathcal{Q}^{n+1}$
Geometric median	$\min \sum t_i$
$\min_x \sum \ x - x_i\ _2$	$(t_i, x - x_i) \in \mathcal{Q}^{n+1}$
Analytic center	$\max \sum t_i$
$\max_x \sum \log(b_i - a_i^T x)$	$(b_i - a_i^T x, 1, t_i) \in K_{\text{exp}}$

## Regression and fitting

Regularized least squares	$\min t + \lambda r$
$\min_w \ Xw - y\ _2^2 + \lambda \ w\ _2^2$	$(0.5, t, Xw - y) \in \mathcal{Q}_r^{m+2}$ $(0.5, r, w) \in \mathcal{Q}_r^{n+2}$
Max likelihood	$\max \sum a_i t_i$
$\max_p p_1^{a_1} \dots p_n^{a_n}$	$(p_i, 1, t_i) \in K_{\text{exp}}$
Logistic cost function	$u + v \leq 1$
$t \geq -\log(1/(1 + e^{-\theta^T x}))$	$(u, 1, -\theta^T x - t) \in K_{\text{exp}}$ $(v, 1, -t) \in K_{\text{exp}}$

## Risk-return

$\Sigma \in \mathbb{R}^{n \times n}$ - covariance, $\Sigma = LL^T$ , $L \in \mathbb{R}^{n \times k}$	
$\max_x \alpha^T x$	$\max_x \alpha^T x$
s.t. $x^T \Sigma x \leq \gamma$	$(\sqrt{\gamma}, L^T x) \in \mathcal{Q}^{k+1}$
$\max_x \alpha^T x - \delta x^T \Sigma x$	$\max_x \alpha^T x - \delta r$ $(0.5, r, L^T x) \in \mathcal{Q}^{k+2}$
Risk plus $x^{1.5}$ impact cost	$t \geq \delta r + \beta \sum u_i$
$t \geq \delta x^T \Sigma x + \beta \sum  x_i ^{3/2}$	$(0.5, r, L^T x) \in \mathcal{Q}^{k+2}$ $(u_i, 1, x_i) \in \mathcal{P}_3^{2/3, 1/3}$
Risk in factor model	$\gamma \geq t + s$
$\gamma \geq x^T (D + F S F^T) x$	$(0.5, t, \sqrt{D} x) \in \mathcal{Q}_r^{n+2}$
$D$ - specific risk (diag.)	$(0.5, s, U^T F^T x) \in \mathcal{Q}_r^{k+2}$
$F \in \mathbb{R}^{n \times k}$ - factor loads	
$S = UU^T$ - factor cov.	