Building large-scale conic optimization models using MOSEK Fusion

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MOSEK

Outline

What's next?

1. Intro on MOSEK
2. Overview of the Fusion API
3. A step-by-step SOCP example
4. Case Study: Quadratic Multi-commodity Min-cost flow
5. Conclusion

MOSEK

A package for large-scale (conic) optimization.

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Algorithms

MOSEK

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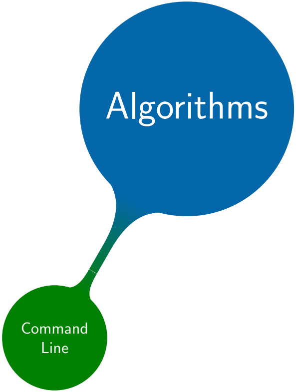

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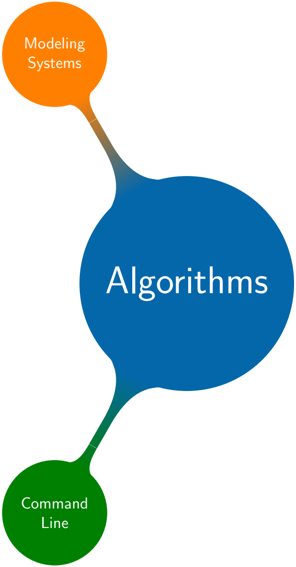

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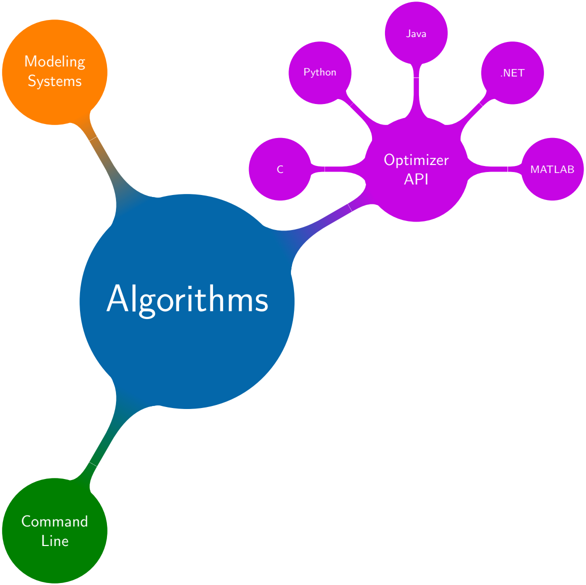

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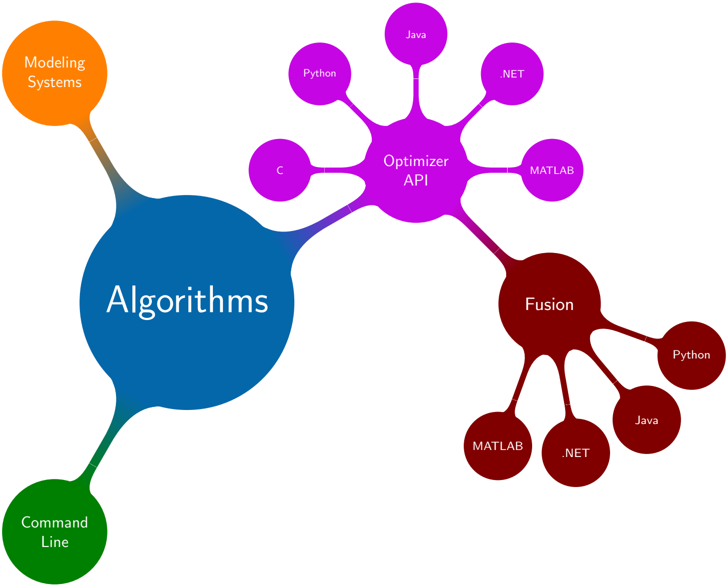

What is Fusion?

The general ideas.

A light-weight object-oriented API for linear conic optimization problems.

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The general ideas.

A light-weight object-oriented API for linear conic optimization problems.

- Minimal memory/time overhead
- Minimal model reformulation/modification
- Improve expressiveness

How Fusion is designed

Representing LCP

$$\min_{x \in \mathbb{R}^n} \quad c^T x$$

$s.t.$

$$A_i x + b_i \in \mathcal{K}_i^{n_i} \in \{\mathbb{R}_+^{n_i}, \mathcal{Q}^{m_i}, \mathcal{Q}_r^{m_i}\} \quad i = 1, \ldots, k$$

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Basic constructs:

```
obj. fun. := ( linear expression , sense)
variable  := ( dimension , domain)
constraint:= ( linear expression , domain)
```

- interface to the solver
- memory manager
- creates its own components
- its components cannot be shared

An SOCP Example

Optimization Model

- interface to the solver
- memory manager
- creates its own components
- its components cannot be shared

```
import mosek
from mosek.fusion import *

M= Model('SOCP example')
```

An SOCP Example

Variables and Constraints

$$x^k \in \mathbb{R}^n_+ \quad k = 1, \ldots, m \quad \Longleftrightarrow \quad X \in \mathbb{R}^{n \times m}_+$$

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```
X = M.variable(NDSet(n,m), Domain.greaterThan(0.))
```

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$$A^k x^k = b^k \quad k = 1, \ldots, m$$

```
for k in range(m):
  M.constraint(Expr.mul(A[k], X.slice([0,k],[n,k+1]),\
                 Domain.equalsTo(b[k]))
```

An SOCP Example

Coupling constraints

$$f_i = \sum d_i x_i^k \leq c_i \qquad i = 1, \ldots, n \iff \qquad \begin{aligned} f - Xd &= 0 \\ f &\leq c \end{aligned}$$

An SOCP Example

Coupling constraints

$$f_i = \sum d_i x_i^k \leq c_i \qquad i = 1, \ldots, n \iff \qquad \begin{aligned} f - Xd &= 0 \\ f &\leq c \end{aligned}$$

```
f=M.variable(n,Domain.lessThan(c))

M.constraint(Expr.sub(f,Expr.mul(X,d)),Domain.equalsTo(0.))
```

An SOCP Example

Objective function

$$\text{minimize} \quad \frac{1}{2}\sum(w_i f_i)^2$$

An SOCP Example

Objective function

$$
\begin{array}{ll}
\text{minimize} & t \\
s.t. & \\
& \left(1, t, w * f\right) \in \mathcal{Q}_r^{2+n}
\end{array}
$$

An SOCP Example

Objective function

$$
\begin{aligned}
&\text{minimize} \quad t \\
&s.t. \\
&\qquad\qquad (1, t, w * f) \in \mathcal{Q}_r^{2+n}
\end{aligned}
$$

```
t= M.variable(1, Domain.unbounded())

M.constraint(Expr.vstack(Expr.constTerm(1.),t,Expr.mulElm(w,f)),\
    Domain.inRotatedQCone())

M.objective(ObjectiveSense.Minimize, t)
```

An SOCP Example

The whole model

```
M= Model('SOCP example')

# X ∈ R_+^{n× m}
X = M.variable(NDSet(n,m), Domain.greaterThan(0.))

#k = 1,...,m
for k in range(m):
    #A^k x^k = b^k
    M.constraint(Expr.mul(A[k], x.slice([0,k],[n,k+1]), Domain.equalsTo(b[k]) )

# f ∈ R^n, f ≤ c
M.variable(n,Domain.lessThan(c))

# f − X d = 0
M.constraint( Expr.sub(f, Expr.mul(X,d) ), Domain.equalsTo(0.))

# t ∈ R
t= M.variable(1, Domain.unbounded())

# (1,t, w∗f) ∈ Q_r^{2+n}
M.constraint(Expr.vstack(Expr.constTerm(1.),t,Expr.mulElm(w,f)),Domain.inRotatedQCone())

# min t
M.objective(ObjectiveSense.Minimize, t)
```

Quadratic Multi-commodity Min-cost Flow

Definition

Given a directed graph G (n_n nodes, n_a arcs, n_o commodities):

- x^k, b^k: the flow and the demand of each commodity
- $A^k = A, \forall k = 1, \ldots, n_o$, with A incidence matrix of G, i.e.

$$AX = B \quad , \quad B = [b^1, b^2, \ldots, b^{n_o}]$$

- f: the total flow vector
- c: arc capacities
- quadratic separable cost, i.e.

$$g(f) = \frac{1}{2} \sum (w_i f_i)^2$$

Quadratic Multi-commodity Min-cost Flow

Fusion code

```
M= Model('Quadratic multi-commodity min-cost flow')

# X ∈ R_+^{na × no}
X= M.variable(NDSet(na,no), Domain.greaterThan(0.) )

# Exploting A,B sparsity
A= Matrix.sparse(nn, na, A_rows, A_cols, A_nnz)
B= Matrix.sparse(nn, no, B_rows, B_cols, B_nnz)

# AX = B
M.constraint(Expr.mul(A, X), Domain.equalsTo(B) )

# 0 ≤ f ≤ c
f= M.variable(na, Domain.lessThan(c) )

# f − X 1_{no} = 0
ones= [1.0 for i in range(no)]
M.constraint(Expr.sub(f, Expr.mul(X, ones)), Domain.equalsTo(0.))

# t ∈ R
t= M.variable(1, Domain.unbounded())

# (1, t, sqrt(w) * f) ∈ Q_r^{2+na}
M.constraint(Expr.vstack(Expr.constTerm(1.),t, Expr.mulElm(w,f), Domain.inRotatedQCone())

# min t
M.objective(ObjectiveSense.Minimize, t)
```

Quadratic Multi-commodity Min-cost Flow

Optimizer API(1)

```
with mosek.Env() as env:
  with env.Task(0,0) as task:
    nx= na*no

    indxf= nx
    nf= na
    indxt=indxf+nf
    nt=1
    indxs=indxt+nt
    ns=1

    numvar= nx+nf+nt+ns
    numcon= na+nn*no

    task.appendcons(numcon)
    task.appendvars(numvar)

    task.putcj( indxt, 1.0)

    #variable bounds setup

    task.putvarboundslice(0, nx, [mosek.boundkey.lo for i in range(nx)],\
      [0. for i in range(nx)], [0. for i in range(nx)])

    task.putvarboundslice(indxf, indxf+nf, [mosek.boundkey.ra for i in range(nf)],\
      [0. for i in range(nf)], cap)

    task.putvarboundslice(indxt, indxt+nt, [mosek.boundkey.lo for i in range(nt)],\
      [0. for i in range(nt)], [0. for i in range(nt)])

    task.putvarbound(indxs, mosek.boundkey.fx, 0.5, 0.5)
```

Quadratic Multi-commodity Min-cost Flow

Optimizer API(2)

```
    for k in range(no):

      task.putaijlist(  A[0], A[1], A[2] )
      A[0]=[ a+nn for a in A[0] ]
      A[1]=[ a+na for a in A[1] ]

      b=[0.0 for i in range(nn) ]
      for (i,j,v)  in B:
        if j==k:
          b[i]= v

      task.putconboundslice(nn*k,nn*(k+1), [mosek.boundkey.fx for i in range(nn)], b,b)

      c_i=[]
      c_j=[]
      c_v=[]
      for i in range(na):
        for j in range(no):
          c_i.append(i+nn*no)
          c_j.append(j*na+i)
          c_v.append(1.0)
          c_i.append(i+nn*no)
          c_j.append(indxf+i)
          c_v.append(-1.0/sqrt(w[i]))

      task.putaijlist( c_i,c_j,c_v)
      task.putconboundslice(nn*no,numcon, [mosek.boundkey.fx for i in range(na)],\
       [0.0 for i in range(na)],\
                                          [0.0 for i in range(na)])

      task.appendcone( mosek.conetype.rquad,0., [indxs,indxt]+ range(indxf,indxf+nf) )
```

Some Computational Experiments

MCMF in urban traffic management

City	n_n	n_a	n_o	F	O
Sioux Falls	24	76	24	0.093 0.059	0.003 0.041
Anaheim	416	914	38	0.210 3.720	0.041 3.189
Barcelona	1020	2838	110	1.011 266.073	0.349 251.037
Winnipeg	1052	2836	147	1.667 198.321	0.509 157.036
Chicago S.	933	2950	387	14.057 228.600	2.915 217.759

Dataset adapted from `http://www.bgu.ac.il/ bargera/tntp/`

Conclusion

and future directions.

We show how Fusion

- introduces a (reasonable) overhead;
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What's ahead?

- C++ interface
- reduce the Fusion/Opt. gap
- more operations and syntactic sugar?

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