Solving the pooling problem using semidefinite programming

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MOSEK ApS

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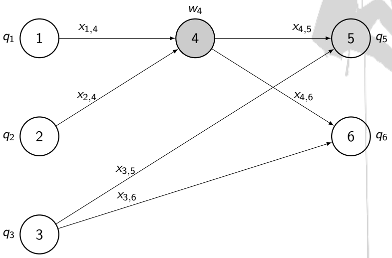
Joint work with Martin S. Andersen, DTU

The pooling problem

- Oil is transported from 3 sources to 2 terminals through a capacitated network.
- Source contamination parameters $\{q_1, q_2, q_3\}$.
- Source 1 and 2 blended at pool 4 with blend quality w_4.
- Terminal quality requirements $\{q_5, q_6\}$.
- Find a flow $\{x_{ij}\}$ and blend w_4 that minimizes transportation cost and satisfies quality requirements.



Haverly network.

Haverly, C. A. Studies of the Behaviour of Recursions for the Pooling Problem, ACM SIGMAP Bull. 1978.

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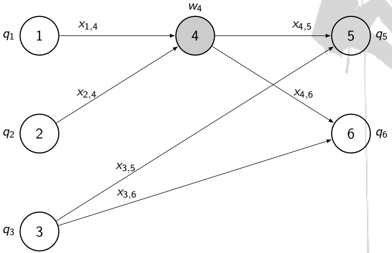

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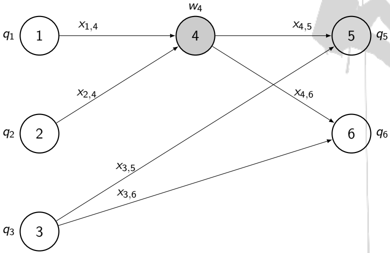

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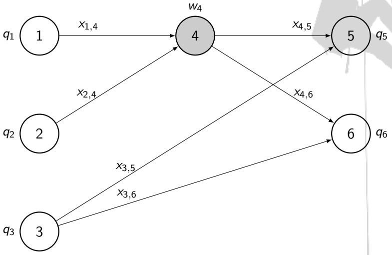

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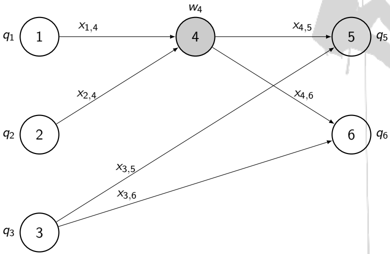

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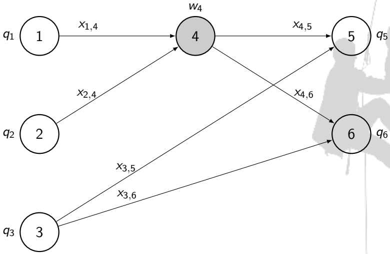
Formulation of optimization problem

- Flow conservation at pools:

$$x_{14} + x_{24} = x_{45} + x_{46}$$



- Defining equation for blend variable:

$$w_4(x_{45} + x_{46}) = q_1 x_{14} + q_2 x_{24}$$

- Quality bounds at terminals:

$$w_4 x_{45} + q_3 x_{35} \leq q_5(x_{45} + x_{35})$$
$$w_4 x_{46} + q_3 x_{36} \leq q_6(x_{46} + x_{36})$$

- Capacity bounds:

$$x_{14} \leq c_1, \quad x_{24} \leq c_2, \quad x_{35}+x_{36} \leq c_3$$
$$x_{45}+x_{46} \leq c_4, \quad x_{35}+x_{45} \leq c_5, \quad x_{36}+x_{46} \leq c_6$$

- Nonnegativity of flow:

$$(x_{14}, x_{24}, x_{35}, x_{36}, x_{45}, x_{46}) \geq 0$$

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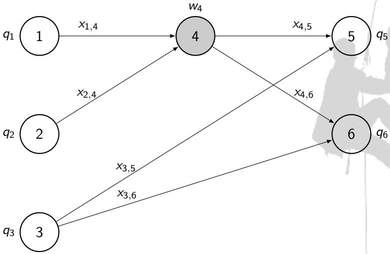

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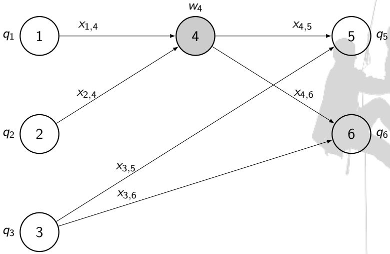

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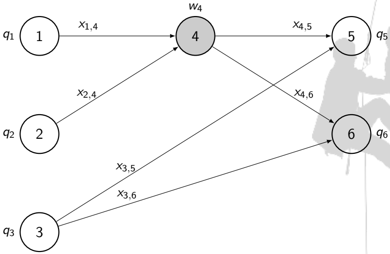

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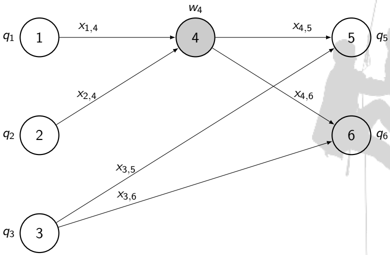

Formulation of optimization problem

Minimum-cost formulation for Haverly

$$
\begin{array}{ll}
\text{minimize} & x_{35} - 9x_{45} - 5x_{36} - 15x_{46} + 6x_{14} + 16x_{24} \\
\text{subject to} & x_{14} + x_{24} = x_{45} + x_{46} \\
& w_4(x_{45} + x_{46}) = q_1 x_{14} + q_2 x_{24} \\
& w_4 x_{45} + q_3 x_{35} \leq q_5(x_{45} + x_{35}) \\
& w_4 x_{46} + q_3 x_{36} \leq q_6(x_{46} + x_{36}) \\
& x_{14} \leq c_1,\; x_{24} \leq c_2 \\
& x_{35} + x_{36} \leq c_3 \\
& x_{45} + x_{46} \leq c_4 \\
& x_{35} + x_{45} \leq c_5 \\
& x_{36} + x_{46} \leq c_6 \\
& (x_{14}, x_{24}, x_{35}, x_{36}, x_{45}, x_{46}) \geq 0
\end{array}
$$

Very difficult to solve (NP hard)!

Introducing semidefinite variables

Let $v := \begin{pmatrix} 1 & x_{35} & x_{45} & x_{36} & x_{46} & x_{14} & x_{24} & w_4 \end{pmatrix}^T$ and define

$$X := \begin{bmatrix}
1 & x_{35} & x_{45} & x_{36} & x_{46} & x_{14} & x_{24} & w_4 \\
x_{35} & x_{35}^2 & x_{35}x_{45} & x_{35}x_{36} & x_{35}x_{46} & x_{35}x_{14} & x_{35}x_{24} & x_{35}w_4 \\
x_{45} & x_{35}x_{45} & x_{45}^2 & x_{45}x_{36} & x_{45}x_{46} & x_{45}x_{14} & x_{45}x_{24} & x_{45}w_4 \\
x_{36} & x_{35}x_{36} & x_{45}x_{36} & x_{36}^2 & x_{36}x_{46} & x_{36}x_{14} & x_{36}x_{24} & x_{36}w_4 \\
x_{46} & x_{35}x_{46} & x_{45}x_{46} & x_{36}x_{46} & x_{46}^2 & x_{46}x_{14} & x_{46}x_{24} & x_{46}w_4 \\
x_{14} & x_{35}x_{14} & x_{45}x_{14} & x_{36}x_{14} & x_{46}x_{14} & x_{14}^2 & x_{14}x_{24} & x_{14}w_4 \\
x_{24} & x_{35}x_{24} & x_{45}x_{24} & x_{36}x_{24} & x_{46}x_{24} & x_{14}x_{24} & x_{24}^2 & x_{24}w_4 \\
w_4 & x_{35}w_4 & x_{45}w_4 & x_{36}w_4 & x_{46}w_4 & x_{14}w_4 & x_{24}w_4 & w_4^2
\end{bmatrix}$$

- Note that $X = vv^T$ with rank 1.
- X contains all 36 monomials up to order 2.
- The pooling problem is equivalent to a *nonconvex SDP* in X.

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- The pooling problem is equivalent to a *nonconvex SDP* in X.

A nonconvex SDP

Treating each monomial as a separate variable, we get a rank-1 SDP:

$$
\begin{aligned}
\text{minimize} \quad & x_{35} - 9x_{45} - 5x_{36} - 15x_{46} + 6x_{14} + 16x_{24} \\
\text{subject to} \quad & \begin{pmatrix}
1 & x_{35} & x_{45} & x_{36} & x_{46} & x_{14} & x_{24} & w_4 \\
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x_{36} & x_{35}x_{36} & x_{45}x_{36} & x_{36}^2 & x_{36}x_{46} & x_{36}x_{14} & x_{36}x_{24} & x_{36}w_4 \\
x_{46} & x_{35}x_{46} & x_{45}x_{46} & x_{36}x_{46} & x_{46}^2 & x_{46}x_{14} & x_{46}x_{24} & x_{46}w_4 \\
x_{14} & x_{35}x_{14} & x_{45}x_{14} & x_{36}x_{14} & x_{46}x_{14} & x_{14}^2 & x_{14}x_{24} & x_{14}w_4 \\
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w_4 & x_{35}w_4 & x_{45}w_4 & x_{36}w_4 & x_{46}w_4 & x_{14}w_4 & x_{24}w_4 & w_4^2
\end{pmatrix} = vv^T \\
& x_{14} + x_{24} = x_{45} + x_{46} \\
& w_4x_{45} + w_4x_{46} = q_1x_{14} + q_2x_{24} \\
& w_4x_{45} + q_3x_{35} \leq q_5(x_{45} + x_{35}) \\
& w_4x_{46} + q_3x_{36} \leq q_6(x_{46} + x_{36}) \\
& x_{14} \leq c_1, \; x_{24} \leq c_2 \\
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& (x_{14}, x_{24}, x_{35}, x_{36}, x_{45}, x_{46}) \geq 0.
\end{aligned}
$$

Using Lasserre relaxations

Method proposed by Frimannslund, El Ghami, Alfaki and Haugland:

- Eliminate equality constraints,

$$\begin{aligned} x_{14} &= \frac{1}{q_1 - q_2} w_4 (x_{45} + x_{46}) - \frac{q_2}{q_1 - q_2} (x_{45} + \frac{1}{2} x_{46}) \\ x_{24} &= -\frac{1}{q_1 - q_2} w_4 (x_{45} + x_{46}) + \frac{q_1}{q_1 - q_2} (x_{45} + \frac{1}{2} x_{46}) \end{aligned}$$

- Tighten relaxation by redundant constraints,

$$\min\{q_1, q_2\} \leq w_4 \leq \max\{q_1, q_2\}$$

- Solve sequence of Lasserre relaxations.

Solving the pooling problem with LMI relaxations, L. Frimmandslund, et. al, 2012.

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\end{aligned}
$$

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Frimannslund's relaxation

$$
\begin{aligned}
\text{minimize} \quad & x_{35} + 12x_{45} - 5x_{36} + 6x_{46} - 5x_{45}w_4 - 5x_{46}w_4 \\
\text{subject to} \quad & \begin{bmatrix} 1 & x_{35} & x_{45} & x_{36} & x_{46} & w_4 \\ x_{35} & x_{35}^2 & x_{35}x_{45} & x_{35}x_{36} & x_{35}x_{46} & x_{35}w_4 \\ x_{45} & x_{35}x_{45} & x_{45}^2 & x_{45}x_{36} & x_{45}x_{46} & x_{45}w_4 \\ x_{36} & x_{35}x_{36} & x_{45}x_{36} & x_{36}^2 & x_{36}x_{46} & x_{36}w_4 \\ x_{46} & x_{35}x_{46} & x_{45}x_{46} & x_{36}x_{46} & x_{46}^2 & x_{46}w_4 \\ w_4 & x_{35}w_4 & x_{45}w_4 & x_{36}w_4 & x_{46}w_4 & w_4^2 \end{bmatrix} \succeq 0 \\
& 0 \leq \tfrac{1}{2}x_{35} + \tfrac{5}{2}x_{45} - x_{45}w_4 \\
& 0 \leq -\tfrac{1}{2}x_{36} + \tfrac{3}{2}x_{46} - x_{46}w_4 \\
& 0 \leq -\tfrac{1}{2}x_{45} - \tfrac{1}{2}x_{46} + \tfrac{1}{2}x_{45}w_4 + \tfrac{1}{2}x_{46}w_4 \leq 1 \\
& 0 \leq \tfrac{3}{2}x_{45} + \tfrac{3}{2}x_{46} - \tfrac{1}{2}x_{45}w_4 - \tfrac{1}{2}x_{46}w_4 \leq 1 \\
& x_{35} + x_{36} \leq 1 \\
& x_{45} + x_{46} \leq 1 \\
& x_{35} + x_{45} \leq (1/3) \\
& x_{36} + x_{46} \leq (2/3) \\
& 1 \leq w_4, \; w_4 \leq 3 \\
& x_{35}, x_{36}, x_{45}, x_{46} \geq 0
\end{aligned}
$$

- 1st order Lasserre relaxation shown.
- Minor suggestion: square bounds instead of adding

$$x_{35}^2 + x_{36}^2 + x_{45}^2 + x_{46}^2 + w_4^2 \leq M.$$

Frimannslund's relaxation

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\begin{aligned}
\text{minimize} \quad & x_{35} + 12x_{45} - 5x_{36} + 6x_{46} - 5x_{45}w_4 - 5x_{46}w_4 \\
\text{subject to} \quad & \begin{bmatrix} 1 & x_{35} & x_{45} & x_{36} & x_{46} & w_4 \\ x_{35} & x_{35}^2 & x_{35}x_{45} & x_{35}x_{36} & x_{35}x_{46} & x_{35}w_4 \\ x_{45} & x_{35}x_{45} & x_{45}^2 & x_{45}x_{36} & x_{45}x_{46} & x_{45}w_4 \\ x_{36} & x_{35}x_{36} & x_{45}x_{36} & x_{36}^2 & x_{36}x_{46} & x_{36}w_4 \\ x_{46} & x_{35}x_{46} & x_{45}x_{46} & x_{36}x_{46} & x_{46}^2 & x_{46}w_4 \\ w_4 & x_{35}w_4 & x_{45}w_4 & x_{36}w_4 & x_{46}w_4 & w_4^2 \end{bmatrix} \succeq 0 \\
& 0 \le \tfrac{1}{2}x_{35} + \tfrac{5}{2}x_{45} - x_{45}w_4 \\
& 0 \le -\tfrac{1}{2}x_{36} + \tfrac{3}{2}x_{46} - x_{46}w_4 \\
& 0 \le -\tfrac{1}{2}x_{45} - \tfrac{1}{2}x_{46} + \tfrac{1}{2}x_{45}w_4 + \tfrac{1}{2}x_{46}w_4 \le 1 \\
& 0 \le \tfrac{3}{2}x_{45} + \tfrac{3}{2}x_{46} - \tfrac{1}{2}x_{45}w_4 - \tfrac{1}{2}x_{46}w_4 \le 1 \\
& (x_{35} + x_{36})^2 \le 1 \\
& (x_{45} + x_{46})^2 \le 1 \\
& (x_{35} + x_{45})^2 \le (1/3) \\
& (x_{36} + x_{46})^2 \le (2/3) \\
& 1 \le w_4, \; w_4^2 \le 3^2 \\
& x_{35}, x_{36}, x_{45}, x_{46} \ge 0
\end{aligned}
$$

- 1st order Lasserre relaxation shown.
- Minor suggestion: square bounds instead of adding

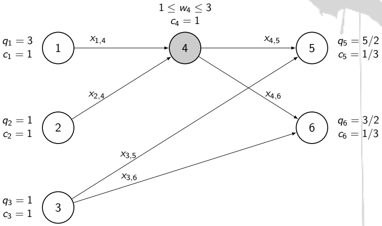
$$x_{35}^2 + x_{36}^2 + x_{45}^2 + x_{46}^2 + w_4^2 \le M.$$

Numerical experiments for Haverly1

$(x_{14}, x_{24}, x_{35}, x_{45}, x_{36}, x_{46}, w_4) = (0, 1/3, 0, 0, 1/3, 1/3, 1)$ found at relaxation order 2.

Problem and solver statistics

order	1	2	3
# LMIs	17	17	17
largest LMI	6	21	56
# vars	37	567	5292
# cons	20	125	461
time (sec)	< 1	< 1	< 1

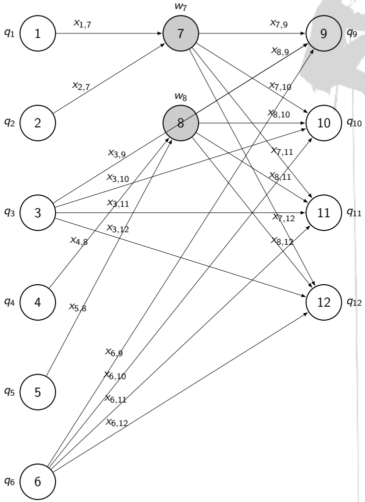


Numerical experiments for Foulds2

- Optimal solution found at relaxation order 2.
- For order 3 we run out of memory.

Problem and solver statistics

order	1	2
# LMIs	41	41
largest LMI	19	190
# vars	230	25745
# cons	189	7314
time (sec)	< 1	103

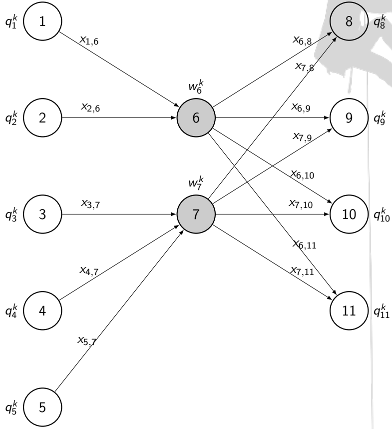


Numerical experiments for Adhya1

- Optimal bound (probably) found at relaxation order 3.
- Feasible solution not recovered due to inaccuracies.

Problem and solver statistics

order	1	2	3
# LMIs	57	57	57
largest LMI	12	78	364
# vars	134	7449	238966
# cons	77	1364	12375
time (sec)	< 1	8	1197



$k = 1, \ldots, 4$

Exploiting sparse structure

Let us map all the monomials we actually need:

$$
\begin{array}{ll}
\text{minimize} & x_{35} + 12x_{45} - 5x_{36} + 6x_{46} - 5x_{45}w_4 - 5x_{46}w_4 \\
\text{subject to} & \begin{bmatrix} 1 & x_{35} & x_{45} & x_{36} & x_{46} & w_4 \\ x_{35} & x_{35}^2 & x_{35}x_{45} & x_{35}x_{36} & x_{35}x_{46} & x_{35}w_4 \\ x_{45} & x_{35}x_{45} & x_{45}^2 & x_{45}x_{36} & x_{45}x_{46} & x_{45}w_4 \\ x_{36} & x_{35}x_{36} & x_{45}x_{36} & x_{36}^2 & x_{36}x_{46} & x_{36}w_4 \\ x_{46} & x_{35}x_{46} & x_{45}x_{46} & x_{36}x_{46} & x_{46}^2 & x_{46}w_4 \\ w_4 & x_{35}w_4 & x_{45}w_4 & x_{36}w_4 & x_{46}w_4 & w_4^2 \end{bmatrix} \succeq 0 \\
& 0 \leq \frac{1}{2}x_{35} + \frac{5}{2}x_{45} - x_{45}w_4 \\
& 0 \leq -\frac{1}{2}x_{36} + \frac{3}{2}x_{46} - x_{46}w_4 \\
& 0 \leq -\frac{1}{2}x_{45} - \frac{1}{2}x_{46} + \frac{1}{2}x_{45}w_4 + \frac{1}{2}x_{46}w_4 \leq 1 \\
& 0 \leq \frac{3}{2}x_{45} + \frac{3}{2}x_{46} - \frac{1}{2}x_{45}w_4 - \frac{1}{2}x_{46}w_4 \leq 1 \\
& (x_{35} + x_{36})^2 \leq 1 \\
& (x_{45} + x_{46})^2 \leq 1 \\
& (x_{35} + x_{45})^2 \leq (1/3)^2 \\
& (x_{36} + x_{46})^2 \leq (2/3)^2 \\
& 1 \leq w_4,\ w_4^2 \leq 3^2 \\
& x_{35}, x_{36}, x_{45}, x_{46} \geq 0
\end{array}
$$

Exploiting sparse structure

Let us map all the monomials we actually need:

$$
\begin{array}{ll}
\text{minimize} & x_{35} + 12x_{45} - 5x_{36} + 6x_{46} - 5x_{45}w_4 - 5x_{46}w_4 \\
\text{subject to} & \begin{bmatrix}
1 & x_{35} & x_{45} & x_{36} & x_{46} & w_4 \\
x_{35} & x_{35}^2 & x_{35}x_{45} & x_{35}x_{36} & x_{35}x_{46} & x_{35}w_4 \\
x_{45} & x_{35}x_{45} & x_{45}^2 & x_{45}x_{36} & x_{45}x_{46} & x_{45}w_4 \\
x_{36} & x_{35}x_{36} & x_{45}x_{36} & x_{36}^2 & x_{36}x_{46} & x_{36}w_4 \\
x_{46} & x_{35}x_{46} & x_{45}x_{46} & x_{36}x_{46} & x_{46}^2 & x_{46}w_4 \\
w_4 & x_{35}w_4 & x_{45}w_4 & x_{36}w_4 & x_{46}w_4 & w_4^2
\end{bmatrix} \succeq 0 \\
& 0 \le \tfrac{1}{2}x_{35} + \tfrac{5}{2}x_{45} - x_{45}w_4 \\
& 0 \le -\tfrac{1}{2}x_{36} + \tfrac{3}{2}x_{46} - x_{46}w_4 \\
& 0 \le -\tfrac{1}{2}x_{45} - \tfrac{1}{2}x_{46} + \tfrac{1}{2}x_{45}w_4 + \tfrac{1}{2}x_{46}w_4 \le 1 \\
& 0 \le \tfrac{3}{2}x_{45} + \tfrac{3}{2}x_{46} - \tfrac{1}{2}x_{45}w_4 - \tfrac{1}{2}x_{46}w_4 \le 1 \\
& (x_{35} + x_{36})^2 \le 1 \\
& (x_{45} + x_{46})^2 \le 1 \\
& (x_{35} + x_{45})^2 \le (1/3)^2 \\
& (x_{36} + x_{46})^2 \le (2/3)^2 \\
& 1 \le w_4,\ w_4^2 \le 3^2 \\
& x_{35}, x_{36}, x_{45}, x_{46} \ge 0
\end{array}
$$

Exploiting sparse structure

Let us map all the monomials we actually need:

$$
\begin{array}{ll}
\text{minimize} & x_{35} + 12x_{45} - 5x_{36} + 6x_{46} - 5x_{45}w_4 - 5x_{46}w_4 \\
\text{subject to} & \begin{bmatrix}
1 & x_{35} & x_{45} & x_{36} & x_{46} & w_4 \\
x_{35} & x_{35}^2 & x_{35}x_{45} & x_{35}x_{36} & x_{35}x_{46} & x_{35}w_4 \\
x_{45} & x_{35}x_{45} & x_{45}^2 & x_{45}x_{36} & x_{45}x_{46} & x_{45}w_4 \\
x_{36} & x_{35}x_{36} & x_{45}x_{36} & x_{36}^2 & x_{36}x_{46} & x_{36}w_4 \\
x_{46} & x_{35}x_{46} & x_{45}x_{46} & x_{36}x_{46} & x_{46}^2 & x_{46}w_4 \\
w_4 & x_{35}w_4 & x_{45}w_4 & x_{36}w_4 & x_{46}w_4 & w_4^2
\end{bmatrix} \succeq 0 \\
& 0 \le \tfrac{1}{2}x_{35} + \tfrac{5}{2}x_{45} - x_{45}w_4 \\
& 0 \le -\tfrac{1}{2}x_{36} + \tfrac{3}{2}x_{46} - x_{46}w_4 \\
& 0 \le -\tfrac{1}{2}x_{45} - \tfrac{1}{2}x_{46} + \tfrac{1}{2}x_{45}w_4 + \tfrac{1}{2}x_{46}w_4 \le 1 \\
& 0 \le \tfrac{3}{2}x_{45} + \tfrac{3}{2}x_{46} - \tfrac{1}{2}x_{45}w_4 - \tfrac{1}{2}x_{46}w_4 \le 1 \\
& (x_{35} + x_{36})^2 \le 1 \\
& (x_{45} + x_{46})^2 \le 1 \\
& (x_{35} + x_{45})^2 \le (1/3)^2 \\
& (x_{36} + x_{46})^2 \le (2/3)^2 \\
& 1 \le w_4,\ w_4^2 \le 3^2 \\
& x_{35}, x_{36}, x_{45}, x_{46} \ge 0
\end{array}
$$

Exploiting sparse structure

Let us map all the monomials we actually need:

$$
\begin{array}{ll}
\text{minimize} & x_{35} + 12x_{45} - 5x_{36} + 6x_{46} - 5x_{45}w_4 - 5x_{46}w_4 \\
\text{subject to} & \begin{bmatrix}
1 & x_{35} & x_{45} & x_{36} & x_{46} & w_4 \\
x_{35} & x_{35}^2 & x_{35}x_{45} & x_{35}x_{36} & x_{35}x_{46} & x_{35}w_4 \\
x_{45} & x_{35}x_{45} & x_{45}^2 & x_{45}x_{36} & x_{45}x_{46} & x_{45}w_4 \\
x_{36} & x_{35}x_{36} & x_{45}x_{36} & x_{36}^2 & x_{36}x_{46} & x_{36}w_4 \\
x_{46} & x_{35}x_{46} & x_{45}x_{46} & x_{36}x_{46} & x_{46}^2 & x_{46}w_4 \\
w_4 & x_{35}w_4 & x_{45}w_4 & x_{36}w_4 & x_{46}w_4 & w_4^2
\end{bmatrix} \succeq 0 \\
& 0 \leq \frac{1}{2}x_{35} + \frac{5}{2}x_{45} - x_{45}w_4 \\
& 0 \leq -\frac{1}{2}x_{36} + \frac{3}{2}x_{46} - x_{46}w_4 \\
& 0 \leq -\frac{1}{2}x_{45} - \frac{1}{2}x_{46} + \frac{1}{2}x_{45}w_4 + \frac{1}{2}x_{46}w_4 \leq 1 \\
& 0 \leq \frac{3}{2}x_{45} + \frac{3}{2}x_{46} - \frac{1}{2}x_{45}w_4 - \frac{1}{2}x_{46}w_4 \leq 1 \\
& (x_{35} + x_{36})^2 \leq 1 \\
& (x_{45} + x_{46})^2 \leq 1 \\
& (x_{35} + x_{45})^2 \leq (1/3)^2 \\
& (x_{36} + x_{46})^2 \leq (2/3)^2 \\
& 1 \leq w_4, \; w_4^2 \leq 3^2 \\
& x_{35}, x_{36}, x_{45}, x_{46} \geq 0
\end{array}
$$

Exploiting sparse structure

Let us map all the monomials we actually need:

$$
\begin{array}{ll}
\text{minimize} & x_{35} + 12x_{45} - 5x_{36} + 6x_{46} - 5x_{45}w_4 - 5x_{46}w_4 \\
\text{subject to} & \begin{bmatrix}
1 & x_{35} & x_{45} & x_{36} & x_{46} & w_4 \\
x_{35} & x_{35}^2 & x_{35}x_{45} & x_{35}x_{36} & x_{35}x_{46} & x_{35}w_4 \\
x_{45} & x_{35}x_{45} & x_{45}^2 & x_{45}x_{36} & x_{45}x_{46} & x_{45}w_4 \\
x_{36} & x_{35}x_{36} & x_{45}x_{36} & x_{36}^2 & x_{36}x_{46} & x_{36}w_4 \\
x_{46} & x_{35}x_{46} & x_{45}x_{46} & x_{36}x_{46} & x_{46}^2 & x_{46}w_4 \\
w_4 & x_{35}w_4 & x_{45}w_4 & x_{36}w_4 & x_{46}w_4 & w_4^2
\end{bmatrix} \succeq 0 \\
& 0 \le \frac{1}{2}x_{35} + \frac{5}{2}x_{45} - x_{45}w_4 \\
& 0 \le -\frac{1}{2}x_{36} + \frac{3}{2}x_{46} - x_{46}w_4 \\
& 0 \le -\frac{1}{2}x_{45} - \frac{1}{2}x_{46} + \frac{1}{2}x_{45}w_4 + \frac{1}{2}x_{46}w_4 \le 1 \\
& 0 \le \frac{3}{2}x_{45} + \frac{3}{2}x_{46} - \frac{1}{2}x_{45}w_4 - \frac{1}{2}x_{46}w_4 \le 1 \\
& (x_{35} + x_{36})^2 \le 1 \\
& (x_{45} + x_{46})^2 \le 1 \\
& (x_{35} + x_{45})^2 \le (1/3)^2 \\
& (x_{36} + x_{46})^2 \le (2/3)^2 \\
& 1 \le w_4,\ w_4^2 \le 3^2 \\
& x_{35}, x_{36}, x_{45}, x_{46} \ge 0
\end{array}
$$

Exploiting sparse structure

Let us map all the monomials we actually need:

$$
\begin{array}{ll}
\text{minimize} & x_{35} + 12x_{45} - 5x_{36} + 6x_{46} - 5x_{45}w_4 - 5x_{46}w_4 \\
\text{subject to} & \begin{bmatrix} 1 & x_{35} & x_{45} & x_{36} & x_{46} & w_4 \\ x_{35} & x_{35}^2 & x_{35}x_{45} & x_{35}x_{36} & x_{35}x_{46} & x_{35}w_4 \\ x_{45} & x_{35}x_{45} & x_{45}^2 & x_{45}x_{36} & x_{45}x_{46} & x_{45}w_4 \\ x_{36} & x_{35}x_{36} & x_{45}x_{36} & x_{36}^2 & x_{36}x_{46} & x_{36}w_4 \\ x_{46} & x_{35}x_{46} & x_{45}x_{46} & x_{36}x_{46} & x_{46}^2 & x_{46}w_4 \\ w_4 & x_{35}w_4 & x_{45}w_4 & x_{36}w_4 & x_{46}w_4 & w_4^2 \end{bmatrix} \succeq 0 \\
& 0 \le \frac{1}{2}x_{35} + \frac{5}{2}x_{45} - x_{45}w_4 \\
& 0 \le -\frac{1}{2}x_{36} + \frac{3}{2}x_{46} - x_{46}w_4 \\
& 0 \le -\frac{1}{2}x_{45} - \frac{1}{2}x_{46} + \frac{1}{2}x_{45}w_4 + \frac{1}{2}x_{46}w_4 \le 1 \\
& 0 \le \frac{3}{2}x_{45} + \frac{3}{2}x_{46} - \frac{1}{2}x_{45}w_4 - \frac{1}{2}x_{46}w_4 \le 1 \\
& (x_{35} + x_{36})^2 \le 1 \\
& (x_{45} + x_{46})^2 \le 1 \\
& (x_{35} + x_{45})^2 \le (1/3)^2 \\
& (x_{36} + x_{46})^2 \le (2/3)^2 \\
& 1 \le w_4,\ w_4^2 \le 3^2 \\
& x_{35}, x_{36}, x_{45}, x_{46} \ge 0
\end{array}
$$

Exploiting sparse structure

Let us map all the monomials we actually need:

$$
\begin{array}{ll}
\text{minimize} & x_{35} + 12x_{45} - 5x_{36} + 6x_{46} - 5x_{45}w_4 - 5x_{46}w_4 \\
\text{subject to} & \begin{bmatrix} 1 & x_{35} & x_{45} & x_{36} & x_{46} & w_4 \\ x_{35} & x_{35}^2 & x_{35}x_{45} & x_{35}x_{36} & x_{35}x_{46} & x_{35}w_4 \\ x_{45} & x_{35}x_{45} & x_{45}^2 & x_{45}x_{36} & x_{45}x_{46} & x_{45}w_4 \\ x_{36} & x_{35}x_{36} & x_{45}x_{36} & x_{36}^2 & x_{36}x_{46} & x_{36}w_4 \\ x_{46} & x_{35}x_{46} & x_{45}x_{46} & x_{36}x_{46} & x_{46}^2 & x_{46}w_4 \\ w_4 & x_{35}w_4 & x_{45}w_4 & x_{36}w_4 & x_{46}w_4 & w_4^2 \end{bmatrix} \succeq 0 \\
& 0 \leq \frac{1}{2}x_{35} + \frac{5}{2}x_{45} - x_{45}w_4 \\
& 0 \leq -\frac{1}{2}x_{36} + \frac{3}{2}x_{46} - x_{46}w_4 \\
& 0 \leq -\frac{1}{2}x_{45} - \frac{1}{2}x_{46} + \frac{1}{2}x_{45}w_4 + \frac{1}{2}x_{46}w_4 \leq 1 \\
& 0 \leq \frac{3}{2}x_{45} + \frac{3}{2}x_{46} - \frac{1}{2}x_{45}w_4 - \frac{1}{2}x_{46}w_4 \leq 1 \\
& (x_{35} + x_{36})^2 \leq 1 \\
& (x_{45} + x_{46})^2 \leq 1 \\
& (x_{35} + x_{45})^2 \leq (1/3)^2 \\
& (x_{36} + x_{46})^2 \leq (2/3)^2 \\
& 1 \leq w_4, \; w_4^2 \leq 3^2 \\
& x_{35}, x_{36}, x_{45}, x_{46} \geq 0
\end{array}
$$

Exploiting sparse structure

Let us map all the monomials we actually need:

$$
\begin{array}{ll}
\text{minimize} & x_{35} + 12x_{45} - 5x_{36} + 6x_{46} - 5x_{45}w_4 - 5x_{46}w_4 \\
\text{subject to} & \begin{bmatrix}
1 & x_{35} & x_{45} & x_{36} & x_{46} & w_4 \\
x_{35} & x_{35}^2 & x_{35}x_{45} & x_{35}x_{36} & x_{35}x_{46} & x_{35}w_4 \\
x_{45} & x_{35}x_{45} & x_{45}^2 & x_{45}x_{36} & x_{45}x_{46} & x_{45}w_4 \\
x_{36} & x_{35}x_{36} & x_{45}x_{36} & x_{36}^2 & x_{36}x_{46} & x_{36}w_4 \\
x_{46} & x_{35}x_{46} & x_{45}x_{46} & x_{36}x_{46} & x_{46}^2 & x_{46}w_4 \\
w_4 & x_{35}w_4 & x_{45}w_4 & x_{36}w_4 & x_{46}w_4 & w_4^2
\end{bmatrix} \succeq 0 \\
& 0 \leq \frac{1}{2}x_{35} + \frac{5}{2}x_{45} - x_{45}w_4 \\
& 0 \leq -\frac{1}{2}x_{36} + \frac{3}{2}x_{46} - x_{46}w_4 \\
& 0 \leq -\frac{1}{2}x_{45} - \frac{1}{2}x_{46} + \frac{1}{2}x_{45}w_4 + \frac{1}{2}x_{46}w_4 \leq 1 \\
& 0 \leq \frac{3}{2}x_{45} + \frac{3}{2}x_{46} - \frac{1}{2}x_{45}w_4 - \frac{1}{2}x_{46}w_4 \leq 1 \\
& (x_{35} + x_{36})^2 \leq 1 \\
& (x_{45} + x_{46})^2 \leq 1 \\
& (x_{35} + x_{45})^2 \leq (1/3)^2 \\
& (x_{36} + x_{46})^2 \leq (2/3)^2 \\
& 1 \leq w_4, \; w_4^2 \leq 3^2 \\
& x_{35}, x_{36}, x_{45}, x_{46} \geq 0
\end{array}
$$

Exploiting sparse structure

Let us map all the monomials we actually need:

$$
\begin{array}{ll}
\text{minimize} & x_{35} + 12x_{45} - 5x_{36} + 6x_{46} - 5x_{45}w_4 - 5x_{46}w_4 \\
\text{subject to} & \begin{bmatrix} 1 & x_{35} & x_{45} & x_{36} & x_{46} & w_4 \\ x_{35} & x_{35}^2 & x_{35}x_{45} & x_{35}x_{36} & x_{35}x_{46} & x_{35}w_4 \\ x_{45} & x_{35}x_{45} & x_{45}^2 & x_{45}x_{36} & x_{45}x_{46} & x_{45}w_4 \\ x_{36} & x_{35}x_{36} & x_{45}x_{36} & x_{36}^2 & x_{36}x_{46} & x_{36}w_4 \\ x_{46} & x_{35}x_{46} & x_{45}x_{46} & x_{36}x_{46} & x_{46}^2 & x_{46}w_4 \\ w_4 & x_{35}w_4 & x_{45}w_4 & x_{36}w_4 & x_{46}w_4 & w_4^2 \end{bmatrix} \succeq 0 \\
& 0 \le \quad \frac{1}{2}x_{35} + \frac{5}{2}x_{45} - x_{45}w_4 \\
& 0 \le -\frac{1}{2}x_{36} + \frac{3}{2}x_{46} - x_{46}w_4 \\
& 0 \le -\frac{1}{2}x_{45} - \frac{1}{2}x_{46} + \frac{1}{2}x_{45}w_4 + \frac{1}{2}x_{46}w_4 \le 1 \\
& 0 \le \quad \frac{3}{2}x_{45} + \frac{3}{2}x_{46} - \frac{1}{2}x_{45}w_4 - \frac{1}{2}x_{46}w_4 \le 1 \\
& (x_{35} + x_{36})^2 \le 1 \\
& (x_{45} + x_{46})^2 \le 1 \\
& (x_{35} + x_{45})^2 \le (1/3)^2 \\
& (x_{36} + x_{46})^2 \le (2/3)^2 \\
& 1 \le w_4, \; w_4^2 \le 3^2 \\
& x_{35}, x_{36}, x_{45}, x_{46} \ge 0
\end{array}
$$

Exploiting sparse structure

Let us map all the monomials we actually need:

$$
\begin{array}{ll}
\text{minimize} & x_{35} + 12x_{45} - 5x_{36} + 6x_{46} - 5x_{45}w_4 - 5x_{46}w_4 \\
\text{subject to} & \begin{bmatrix}
1 & x_{35} & x_{45} & x_{36} & x_{46} & w_4 \\
x_{35} & x_{35}^2 & x_{35}x_{45} & x_{35}x_{36} & x_{35}x_{46} & x_{35}w_4 \\
x_{45} & x_{35}x_{45} & x_{45}^2 & x_{45}x_{36} & x_{45}x_{46} & x_{45}w_4 \\
x_{36} & x_{35}x_{36} & x_{45}x_{36} & x_{36}^2 & x_{36}x_{46} & x_{36}w_4 \\
x_{46} & x_{35}x_{46} & x_{45}x_{46} & x_{36}x_{46} & x_{46}^2 & x_{46}w_4 \\
w_4 & x_{35}w_4 & x_{45}w_4 & x_{36}w_4 & x_{46}w_4 & w_4^2
\end{bmatrix} \succeq 0 \\
& 0 \leq \frac{1}{2}x_{35} + \frac{5}{2}x_{45} - x_{45}w_4 \\
& 0 \leq -\frac{1}{2}x_{36} + \frac{3}{2}x_{46} - x_{46}w_4 \\
& 0 \leq -\frac{1}{2}x_{45} - \frac{1}{2}x_{46} + \frac{1}{2}x_{45}w_4 + \frac{1}{2}x_{46}w_4 \leq 1 \\
& 0 \leq \frac{3}{2}x_{45} + \frac{3}{2}x_{46} - \frac{1}{2}x_{45}w_4 - \frac{1}{2}x_{46}w_4 \leq 1 \\
& (x_{35} + x_{36})^2 \leq 1 \\
& (x_{45} + x_{46})^2 \leq 1 \\
& (x_{35} + x_{45})^2 \leq (1/3)^2 \\
& (x_{36} + x_{46})^2 \leq (2/3)^2 \\
& 1 \leq w_4,\ w_4^2 \leq 3^2 \\
& x_{35}, x_{36}, x_{45}, x_{46} \geq 0
\end{array}
$$

Exploiting sparse structure

Let us map all the monomials we actually need:

$$
\begin{array}{ll}
\text{minimize} & x_{35} + 12x_{45} - 5x_{36} + 6x_{46} - 5x_{45}w_4 - 5x_{46}w_4 \\
\text{subject to} & \begin{bmatrix} 1 & x_{35} & x_{45} & x_{36} & x_{46} & w_4 \\ x_{35} & x_{35}^2 & x_{35}x_{45} & x_{35}x_{36} & x_{35}x_{46} & x_{35}w_4 \\ x_{45} & x_{35}x_{45} & x_{45}^2 & x_{45}x_{36} & x_{45}x_{46} & x_{45}w_4 \\ x_{36} & x_{35}x_{36} & x_{45}x_{36} & x_{36}^2 & x_{36}x_{46} & x_{36}w_4 \\ x_{46} & x_{35}x_{46} & x_{45}x_{46} & x_{36}x_{46} & x_{46}^2 & x_{46}w_4 \\ w_4 & x_{35}w_4 & x_{45}w_4 & x_{36}w_4 & x_{46}w_4 & w_4^2 \end{bmatrix} \succeq 0 \\
& 0 \leq \frac{1}{2}x_{35} + \frac{5}{2}x_{45} - x_{45}w_4 \\
& 0 \leq -\frac{1}{2}x_{36} + \frac{3}{2}x_{46} - x_{46}w_4 \\
& 0 \leq -\frac{1}{2}x_{45} - \frac{1}{2}x_{46} + \frac{1}{2}x_{45}w_4 + \frac{1}{2}x_{46}w_4 \leq 1 \\
& 0 \leq \frac{3}{2}x_{45} + \frac{3}{2}x_{46} - \frac{1}{2}x_{45}w_4 - \frac{1}{2}x_{46}w_4 \leq 1 \\
& (x_{35} + x_{36})^2 \leq 1 \\
& (x_{45} + x_{46})^2 \leq 1 \\
& (x_{35} + x_{45})^2 \leq (1/3)^2 \\
& (x_{36} + x_{46})^2 \leq (2/3)^2 \\
& 1 \leq w_4,\ w_4^2 \leq 3^2 \\
& x_{35}, x_{36}, x_{45}, x_{46} \geq 0
\end{array}
$$

Exploiting sparse structure

Let us map all the monomials we actually need:

$$
\begin{array}{ll}
\text{minimize} & x_{35} + 12x_{45} - 5x_{36} + 6x_{46} - 5x_{45}w_4 - 5x_{46}w_4 \\
\text{subject to} & \begin{bmatrix} 1 & x_{35} & x_{45} & x_{36} & x_{46} & w_4 \\ x_{35} & x_{35}^2 & x_{35}x_{45} & x_{35}x_{36} & x_{35}x_{46} & x_{35}w_4 \\ x_{45} & x_{35}x_{45} & x_{45}^2 & x_{45}x_{36} & x_{45}x_{46} & x_{45}w_4 \\ x_{36} & x_{35}x_{36} & x_{45}x_{36} & x_{36}^2 & x_{36}x_{46} & x_{36}w_4 \\ x_{46} & x_{35}x_{46} & x_{45}x_{46} & x_{36}x_{46} & x_{46}^2 & x_{46}w_4 \\ w_4 & x_{35}w_4 & x_{45}w_4 & x_{36}w_4 & x_{46}w_4 & w_4^2 \end{bmatrix} \succeq 0 \\
& 0 \leq \quad \frac{1}{2}x_{35} + \frac{5}{2}x_{45} - x_{45}w_4 \\
& 0 \leq -\frac{1}{2}x_{36} + \frac{3}{2}x_{46} - x_{46}w_4 \\
& 0 \leq -\frac{1}{2}x_{45} - \frac{1}{2}x_{46} + \frac{1}{2}x_{45}w_4 + \frac{1}{2}x_{46}w_4 \leq 1 \\
& 0 \leq \quad \frac{3}{2}x_{45} + \frac{3}{2}x_{46} - \frac{1}{2}x_{45}w_4 - \frac{1}{2}x_{46}w_4 \leq 1 \\
& (x_{35} + x_{36})^2 \leq 1 \\
& (x_{45} + x_{46})^2 \leq 1 \\
& (x_{35} + x_{45})^2 \leq (1/3)^2 \\
& (x_{36} + x_{46})^2 \leq (2/3)^2 \\
& 1 \leq w_4, \; w_4^2 \leq 3^2 \\
& x_{35}, x_{36}, x_{45}, x_{46} \geq 0
\end{array}
$$

Exploiting sparse structure

Let us map all the monomials we actually need:

$$
\begin{array}{ll}
\text{minimize} & x_{35} + 12x_{45} - 5x_{36} + 6x_{46} - 5x_{45}w_4 - 5x_{46}w_4 \\
\text{subject to} & \begin{bmatrix}
1 & x_{35} & x_{45} & x_{36} & x_{46} & w_4 \\
x_{35} & x_{35}^2 & x_{35}x_{45} & x_{35}x_{36} & x_{35}x_{46} & x_{35}w_4 \\
x_{45} & x_{35}x_{45} & x_{45}^2 & x_{45}x_{36} & x_{45}x_{46} & x_{45}w_4 \\
x_{36} & x_{35}x_{36} & x_{45}x_{36} & x_{36}^2 & x_{36}x_{46} & x_{36}w_4 \\
x_{46} & x_{35}x_{46} & x_{45}x_{46} & x_{36}x_{46} & x_{46}^2 & x_{46}w_4 \\
w_4 & x_{35}w_4 & x_{45}w_4 & x_{36}w_4 & x_{46}w_4 & w_4^2
\end{bmatrix} \succeq 0 \\
& 0 \le \frac{1}{2}x_{35} + \frac{5}{2}x_{45} - x_{45}w_4 \\
& 0 \le -\frac{1}{2}x_{36} + \frac{3}{2}x_{46} - x_{46}w_4 \\
& 0 \le -\frac{1}{2}x_{45} - \frac{1}{2}x_{46} + \frac{1}{2}x_{45}w_4 + \frac{1}{2}x_{46}w_4 \le 1 \\
& 0 \le \frac{3}{2}x_{45} + \frac{3}{2}x_{46} - \frac{1}{2}x_{45}w_4 - \frac{1}{2}x_{46}w_4 \le 1 \\
& (x_{35} + x_{36})^2 \le 1 \\
& (x_{45} + x_{46})^2 \le 1 \\
& (x_{35} + x_{45})^2 \le (1/3)^2 \\
& (x_{36} + x_{46})^2 \le (2/3)^2 \\
& 1 \le w_4,\ w_4^2 \le 3^2 \\
& x_{35}, x_{36}, x_{45}, x_{46} \ge 0
\end{array}
$$

Chordal embedding

$$X = \begin{bmatrix} 1 & x_{35} & x_{45} & x_{36} & x_{46} & w_4 \\ x_{35} & x_{35}^2 & x_{35}x_{45} & x_{35}x_{36} & x_{35}x_{46} & x_{35}w_4 \\ x_{45} & x_{35}x_{45} & x_{45}^2 & x_{45}x_{36} & x_{45}x_{46} & x_{45}w_4 \\ x_{36} & x_{35}x_{36} & x_{45}x_{36} & x_{36}^2 & x_{36}x_{46} & x_{36}w_4 \\ x_{46} & x_{35}x_{46} & x_{45}x_{46} & x_{36}x_{46} & x_{46}^2 & x_{46}w_4 \\ w_4 & x_{35}w_4 & x_{45}w_4 & x_{36}w_4 & x_{46}w_4 & w_4^2 \end{bmatrix} \succeq 0$$

- In a chordal embedding we add $x_{45}x_{36}$.
- For the chordal matrix we identify the cliques:

$$I_1 = \{1, x_{35}, x_{45}, x_{36}, w_4\},\ I_2 = \{1, x_{45}, x_{36}, x_{46}, w_4\}.$$

- Semidefinite matrix completion:

$$X_{I_1,I_1} \succeq 0,\ X_{I_2,I_2} \succeq 0 \quad \Longleftrightarrow \quad \exists x_{35}x_{46} \ : \ X \succeq 0$$

- I.e., $x_{35}x_{46}$ can be eliminated.

Exploiting "Correlative sparsity pattern", Waki et. al, Lasserre, Mevissen.

Chordal embedding

$$X = \begin{bmatrix} 1 & x_{35} & x_{45} & x_{36} & x_{46} & w_4 \\ x_{35} & x_{35}^2 & x_{35}x_{45} & x_{35}x_{36} & x_{35}x_{46} & x_{35}w_4 \\ x_{45} & x_{35}x_{45} & x_{45}^2 & x_{45}x_{36} & x_{45}x_{46} & x_{45}w_4 \\ x_{36} & x_{35}x_{36} & x_{45}x_{36} & x_{36}^2 & x_{36}x_{46} & x_{36}w_4 \\ x_{46} & x_{35}x_{46} & x_{45}x_{46} & x_{36}x_{46} & x_{46}^2 & x_{46}w_4 \\ w_4 & x_{35}w_4 & x_{45}w_4 & x_{36}w_4 & x_{46}w_4 & w_4^2 \end{bmatrix} \succeq 0$$

- In a chordal embedding we add $x_{45}x_{36}$.
- For the chordal matrix we identify the cliques:

$$I_1 = \{1, x_{35}, x_{45}, x_{36}, w_4\}, \; I_2 = \{1, x_{45}, x_{36}, x_{46}, w_4\}.$$

- Semidefinite matrix completion:

$$X_{I_1,I_1} \succeq 0, \; X_{I_2,I_2} \succeq 0 \quad \Longleftrightarrow \quad \exists x_{35}x_{46} \; : \; X \succeq 0$$

- I.e., $x_{35}x_{46}$ can be eliminated.

Exploiting "Correlative sparsity pattern", Waki et. al, Lasserre, Mevissen.

Chordal embedding

$$X = \begin{bmatrix} 1 & x_{35} & x_{45} & x_{36} & x_{46} & w_4 \\ x_{35} & x_{35}^2 & x_{35}x_{45} & x_{35}x_{36} & x_{35}x_{46} & x_{35}w_4 \\ x_{45} & x_{35}x_{45} & x_{45}^2 & x_{45}x_{36} & x_{45}x_{46} & x_{45}w_4 \\ x_{36} & x_{35}x_{36} & x_{45}x_{36} & x_{36}^2 & x_{36}x_{46} & x_{36}w_4 \\ x_{46} & x_{35}x_{46} & x_{45}x_{46} & x_{36}x_{46} & x_{46}^2 & x_{46}w_4 \\ w_4 & x_{35}w_4 & x_{45}w_4 & x_{36}w_4 & x_{46}w_4 & w_4^2 \end{bmatrix} \succeq 0$$

- In a chordal embedding we add $x_{45}x_{36}$.
- For the chordal matrix we identify the cliques:

$$I_1 = \{1, x_{35}, x_{45}, x_{36}, w_4\}, \; I_2 = \{1, x_{45}, x_{36}, x_{46}, w_4\}.$$

- Semidefinite matrix completion:

$$X_{I_1,I_1} \succeq 0, \; X_{I_2,I_2} \succeq 0 \quad \Longleftrightarrow \quad \exists x_{35}x_{46} \; : \; X \succeq 0$$

- I.e., $x_{35}x_{46}$ can be eliminated.

Exploiting "Correlative sparsity pattern", Waki et. al, Lasserre, Mevissen.

Chordal embedding

$$X = \begin{bmatrix} 1 & x_{35} & x_{45} & x_{36} & x_{46} & w_4 \\ x_{35} & x_{35}^2 & x_{35}x_{45} & x_{35}x_{36} & x_{35}x_{46} & x_{35}w_4 \\ x_{45} & x_{35}x_{45} & x_{45}^2 & x_{45}x_{36} & x_{45}x_{46} & x_{45}w_4 \\ x_{36} & x_{35}x_{36} & x_{45}x_{36} & x_{36}^2 & x_{36}x_{46} & x_{36}w_4 \\ x_{46} & x_{35}x_{46} & x_{45}x_{46} & x_{36}x_{46} & x_{46}^2 & x_{46}w_4 \\ w_4 & x_{35}w_4 & x_{45}w_4 & x_{36}w_4 & x_{46}w_4 & w_4^2 \end{bmatrix} \succeq 0$$

- In a chordal embedding we add $x_{45}x_{36}$.
- For the chordal matrix we identify the cliques:

$$I_1 = \{1, x_{35}, x_{45}, x_{36}, w_4\},\ I_2 = \{1, x_{45}, x_{36}, x_{46}, w_4\}.$$

- Semidefinite matrix completion:

$$X_{I_1,I_1} \succeq 0,\ X_{I_2,I_2} \succeq 0 \quad \iff \quad \exists x_{35}x_{46} \ : \ X \succeq 0$$

- I.e., $x_{35}x_{46}$ can be eliminated.

Exploiting "Correlative sparsity pattern", Waki et. al, Lasserre, Mevissen.

Chordal relaxation for Foulds2

- Optimal solution found at relaxation order 2 for both versions.
- For order 3 we run out of memory for both versions.

Standard Lasserre relaxations

order	1	2
# LMIs	41	41
largest LMI	19	190
# vars	230	25745
# cons	189	7314
time (sec)	< 1	103

Chordal Lasserre relaxations

order	1	2
# LMIs	41	41
largest LMI	13	91
# vars	230	14828
# cons	189	7314
time (sec)	< 1	25

Chordal relaxation for Adhya1

- The 2nd order chordal relaxation is weaker than the dense version.
- Optimal bound found at relaxation order 3 for both versions.
- The solution to the chordal version is feasible.

Standard Lasserre relaxations

order	1	2	3
# LMIs	57	57	57
largest LMI	12	78	364
# vars	134	7449	238966
# cons	77	1364	12375
time (sec)	< 1	8	1197

Chordal Lasserre relaxations

order	1	2	3
# LMIs	59	59	59
largest LMI	10	55	220
# vars	201	6230	114710
# cons	77	1364	12375
time (sec)	< 1	3	218

Tighter relaxations

$$X := \begin{bmatrix} 1 & x_{35} & x_{45} & x_{36} & x_{46} & x_{14} & x_{24} & w_4 \\ x_{35} & x_{35}^2 & x_{35}x_{45} & x_{35}x_{36} & x_{35}x_{46} & x_{35}x_{14} & x_{35}x_{24} & x_{35}w_4 \\ x_{45} & x_{35}x_{45} & x_{45}^2 & x_{45}x_{36} & x_{45}x_{46} & x_{45}x_{14} & x_{45}x_{24} & x_{45}w_4 \\ x_{36} & x_{35}x_{36} & x_{45}x_{36} & x_{36}^2 & x_{36}x_{46} & x_{36}x_{14} & x_{36}x_{24} & x_{36}w_4 \\ x_{46} & x_{35}x_{46} & x_{45}x_{46} & x_{36}x_{46} & x_{46}^2 & x_{46}x_{14} & x_{46}x_{24} & x_{46}w_4 \\ x_{14} & x_{35}x_{14} & x_{45}x_{14} & x_{36}x_{14} & x_{46}x_{14} & x_{14}^2 & x_{14}x_{24} & x_{14}w_4 \\ x_{24} & x_{35}x_{24} & x_{45}x_{24} & x_{36}x_{24} & x_{46}x_{24} & x_{14}x_{24} & x_{24}^2 & x_{24}w_4 \\ w_4 & x_{35}w_4 & x_{45}w_4 & x_{36}w_4 & x_{46}w_4 & x_{14}w_4 & x_{24}w_4 & w_4^2 \end{bmatrix} \succeq 0.$$

- Since $(x_{35}, x_{45}, x_{36}, x_{46}, x_{14}, x_{24}, w_4) \geq 0$ then $X_{ij} \geq 0$.
- X is *doubly nonnegative.*
- Adding some of the constraints $X_{ij} \geq 0$ is very cheap, but strengthens the relaxation.
- Adding all the constraints $X_{ij} \geq 0$ is still cheaper than increasing the relaxation order.

Tighter relaxations

$$X := \begin{bmatrix} 1 & x_{35} & x_{45} & x_{36} & x_{46} & x_{14} & x_{24} & w_4 \\ x_{35} & x_{35}^2 & x_{35}x_{45} & x_{35}x_{36} & x_{35}x_{46} & x_{35}x_{14} & x_{35}x_{24} & x_{35}w_4 \\ x_{45} & x_{35}x_{45} & x_{45}^2 & x_{45}x_{36} & x_{45}x_{46} & x_{45}x_{14} & x_{45}x_{24} & x_{45}w_4 \\ x_{36} & x_{35}x_{36} & x_{45}x_{36} & x_{36}^2 & x_{36}x_{46} & x_{36}x_{14} & x_{36}x_{24} & x_{36}w_4 \\ x_{46} & x_{35}x_{46} & x_{45}x_{46} & x_{36}x_{46} & x_{46}^2 & x_{46}x_{14} & x_{46}x_{24} & x_{46}w_4 \\ x_{14} & x_{35}x_{14} & x_{45}x_{14} & x_{36}x_{14} & x_{46}x_{14} & x_{14}^2 & x_{14}x_{24} & x_{14}w_4 \\ x_{24} & x_{35}x_{24} & x_{45}x_{24} & x_{36}x_{24} & x_{46}x_{24} & x_{14}x_{24} & x_{24}^2 & x_{24}w_4 \\ w_4 & x_{35}w_4 & x_{45}w_4 & x_{36}w_4 & x_{46}w_4 & x_{14}w_4 & x_{24}w_4 & w_4^2 \end{bmatrix} \succeq 0.$$

- Since $(x_{35}, x_{45}, x_{36}, x_{46}, x_{14}, x_{24}, w_4) \geq 0$ then $X_{ij} \geq 0$.
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- Adding all the constraints $X_{ij} \geq 0$ is still cheaper than increasing the relaxation order.

Tighter relaxations

$$X := \begin{bmatrix} 1 & x_{35} & x_{45} & x_{36} & x_{46} & x_{14} & x_{24} & w_4 \\ x_{35} & x_{35}^2 & x_{35}x_{45} & x_{35}x_{36} & x_{35}x_{46} & x_{35}x_{14} & x_{35}x_{24} & x_{35}w_4 \\ x_{45} & x_{35}x_{45} & x_{45}^2 & x_{45}x_{36} & x_{45}x_{46} & x_{45}x_{14} & x_{45}x_{24} & x_{45}w_4 \\ x_{36} & x_{35}x_{36} & x_{45}x_{36} & x_{36}^2 & x_{36}x_{46} & x_{36}x_{14} & x_{36}x_{24} & x_{36}w_4 \\ x_{46} & x_{35}x_{46} & x_{45}x_{46} & x_{36}x_{46} & x_{46}^2 & x_{46}x_{14} & x_{46}x_{24} & x_{46}w_4 \\ x_{14} & x_{35}x_{14} & x_{45}x_{14} & x_{36}x_{14} & x_{46}x_{14} & x_{14}^2 & x_{14}x_{24} & x_{14}w_4 \\ x_{24} & x_{35}x_{24} & x_{45}x_{24} & x_{36}x_{24} & x_{46}x_{24} & x_{14}x_{24} & x_{24}^2 & x_{24}w_4 \\ w_4 & x_{35}w_4 & x_{45}w_4 & x_{36}w_4 & x_{46}w_4 & x_{14}w_4 & x_{24}w_4 & w_4^2 \end{bmatrix} \succeq 0.$$

- Since $(x_{35}, x_{45}, x_{36}, x_{46}, x_{14}, x_{24}, w_4) \geq 0$ then $X_{ij} \geq 0$.
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- Adding some of the constraints $X_{ij} \geq 0$ is very cheap, but strengthens the relaxation.
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Tighter relaxations

$$X := \begin{bmatrix} 1 & x_{35} & x_{45} & x_{36} & x_{46} & x_{14} & x_{24} & w_4 \\ x_{35} & x_{35}^2 & x_{35}x_{45} & x_{35}x_{36} & x_{35}x_{46} & x_{35}x_{14} & x_{35}x_{24} & x_{35}w_4 \\ x_{45} & x_{35}x_{45} & x_{45}^2 & x_{45}x_{36} & x_{45}x_{46} & x_{45}x_{14} & x_{45}x_{24} & x_{45}w_4 \\ x_{36} & x_{35}x_{36} & x_{45}x_{36} & x_{36}^2 & x_{36}x_{46} & x_{36}x_{14} & x_{36}x_{24} & x_{36}w_4 \\ x_{46} & x_{35}x_{46} & x_{45}x_{46} & x_{36}x_{46} & x_{46}^2 & x_{46}x_{14} & x_{46}x_{24} & x_{46}w_4 \\ x_{14} & x_{35}x_{14} & x_{45}x_{14} & x_{36}x_{14} & x_{46}x_{14} & x_{14}^2 & x_{14}x_{24} & x_{14}w_4 \\ x_{24} & x_{35}x_{24} & x_{45}x_{24} & x_{36}x_{24} & x_{46}x_{24} & x_{14}x_{24} & x_{24}^2 & x_{24}w_4 \\ w_4 & x_{35}w_4 & x_{45}w_4 & x_{36}w_4 & x_{46}w_4 & x_{14}w_4 & x_{24}w_4 & w_4^2 \end{bmatrix} \succeq 0.$$

- Since $(x_{35}, x_{45}, x_{36}, x_{46}, x_{14}, x_{24}, w_4) \geq 0$ then $X_{ij} \geq 0$.
- X is *doubly nonnegative*.
- Adding some of the constraints $X_{ij} \geq 0$ is very cheap, but strengthens the relaxation.
- Adding all the constraints $X_{ij} \geq 0$ is still cheaper than increasing the relaxation order.

Other ideas

Reconsider that first minimum-cost formulation:

$$
\begin{array}{ll}
\text{minimize} & x_{35} - 9x_{45} - 5x_{36} - 15x_{46} + 6x_{14} + 16x_{24} \\
\text{subject to} & x_{14} + x_{24} = x_{45} + x_{46} \\
& w_4(x_{45} + x_{46}) = q_1 x_{14} + q_2 x_{24} \\
& w_4 x_{45} + q_3 x_{35} \leq q_5(x_{45} + x_{35}) \\
& w_4 x_{46} + q_3 x_{36} \leq q_6(x_{46} + x_{36}) \\
& x_{14} \leq c_1, \; x_{24} \leq c_2 \\
& x_{35} + x_{36} \leq c_3 \\
& x_{45} + x_{46} \leq c_4 \\
& x_{35} + x_{45} \leq c_5 \\
& x_{36} + x_{46} \leq c_6 \\
& (x_{14}, x_{24}, x_{35}, x_{36}, x_{45}, x_{46}) \geq 0
\end{array}
$$

- Keep equality constraints to get sparser (but larger) LMIs.
- Specialize the Lasserre relaxations for bilinear problems.
- Break up cliques by adding simple redundant constraints.

Other ideas

Reconsider that first minimum-cost formulation:

$$
\begin{array}{ll}
\text{minimize} & x_{35} - 9x_{45} - 5x_{36} - 15x_{46} + 6x_{14} + 16x_{24} \\
\text{subject to} & x_{14} + x_{24} = x_{45} + x_{46} \\
& w_4(x_{45} + x_{46}) = q_1 x_{14} + q_2 x_{24} \\
& w_4 x_{45} + q_3 x_{35} \leq q_5(x_{45} + x_{35}) \\
& w_4 x_{46} + q_3 x_{36} \leq q_6(x_{46} + x_{36}) \\
& x_{14} \leq c_1,\ x_{24} \leq c_2 \\
& x_{35} + x_{36} \leq c_3 \\
& x_{45} + x_{46} \leq c_4 \\
& x_{35} + x_{45} \leq c_5 \\
& x_{36} + x_{46} \leq c_6 \\
& (x_{14}, x_{24}, x_{35}, x_{36}, x_{45}, x_{46}) \geq 0
\end{array}
$$

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Other ideas

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$$
\begin{array}{ll}
\text{minimize} & x_{35} - 9x_{45} - 5x_{36} - 15x_{46} + 6x_{14} + 16x_{24} \\
\text{subject to} & x_{14} + x_{24} = x_{45} + x_{46} \\
& w_4(x_{45} + x_{46}) = q_1 x_{14} + q_2 x_{24} \\
& w_4 x_{45} + q_3 x_{35} \leq q_5(x_{45} + x_{35}) \\
& w_4 x_{46} + q_3 x_{36} \leq q_6(x_{46} + x_{36}) \\
& x_{14} \leq c_1,\ x_{24} \leq c_2 \\
& x_{35} + x_{36} \leq c_3 \\
& x_{45} + x_{46} \leq c_4 \\
& x_{35} + x_{45} \leq c_5 \\
& x_{36} + x_{46} \leq c_6 \\
& (x_{14}, x_{24}, x_{35}, x_{36}, x_{45}, x_{46}) \geq 0
\end{array}
$$

- Keep equality constraints to get sparser (but larger) LMIs.
- Specialize the Lasserre relaxations for bilinear problems.
- Break up cliques by adding simple redundant constraints.

Conclusions

- Exploiting chordal structure gives a noticable improvement, but we can still only solve toy problems.
- The problems are quite difficult to solve with some inherent ill-posedness.
- Scaling of the models is crucial for the solver.
- The relaxations are interesting test problems, because they contain many small LMIs.
- Solving a bilinear problem as a general polynomial optimization problem seems too generic.

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